7. CAPM and Multifactor Models

- CAPM Assumptions
  - Many restrictive assumptions, but most relaxed in Arbitrage Pricing Theory.
  - APT holds for div portfolios and almost surely for assets.

- CAPM Implications
  - Optimal (tangent) portfolio is the market portfolio.
  - No portfolio has higher return-to-variability ratio than the market portfolio.
  - Capital Market Line is the name for the "best" CAL.
  - Expected returns on assets and portfolios satisfy: $E(r_i) = r_f + b_i \times [E(r_M) - r_f]$
  - Security market line describes CAPM equation.

- CAPM regression model
  - Formalizes total risk = systematic risk + unsystematic risk
  - CAPM regression is simply: $(r_i - r_f) = a_i + ß_i(r_m - r_f) + e_i$

- Multifactor Models
  - Empirically, may better explain returns.

CAPM

- CAPM - An equilibrium model (D=S) based on mean-variance analysis.
  - Model provides measure of systematic risk that applies to all assets.
  - Nobody believes CAPM literally, but its simple and “works” pretty good.
  - Assume investors are homogeneous, mean-var optimizers and price takers.
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- CAPM results:
  - All investors (you, me, Dave) hold same (tangent) portfolio of risky assets.
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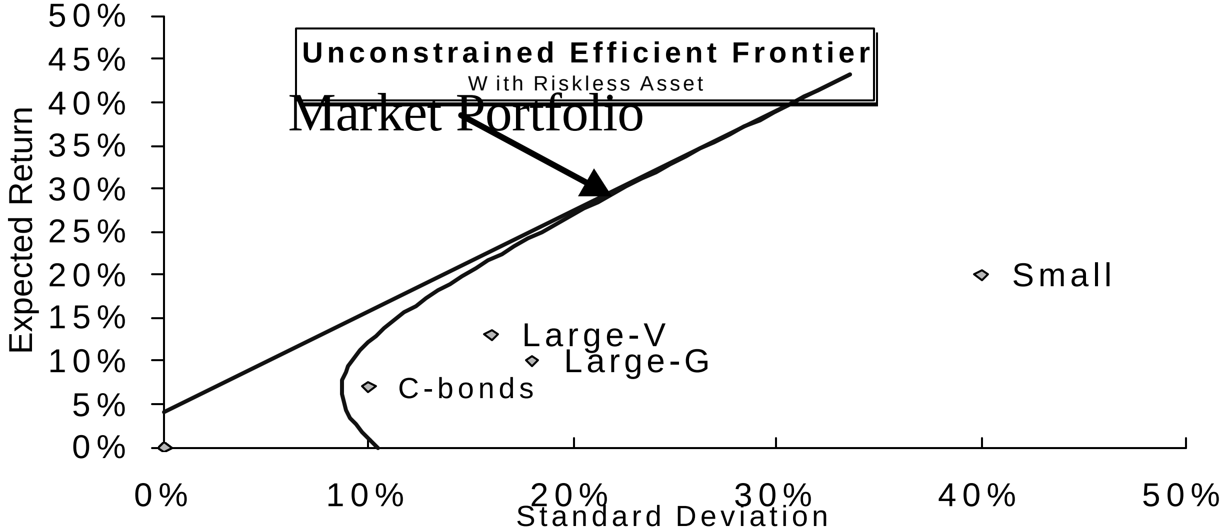
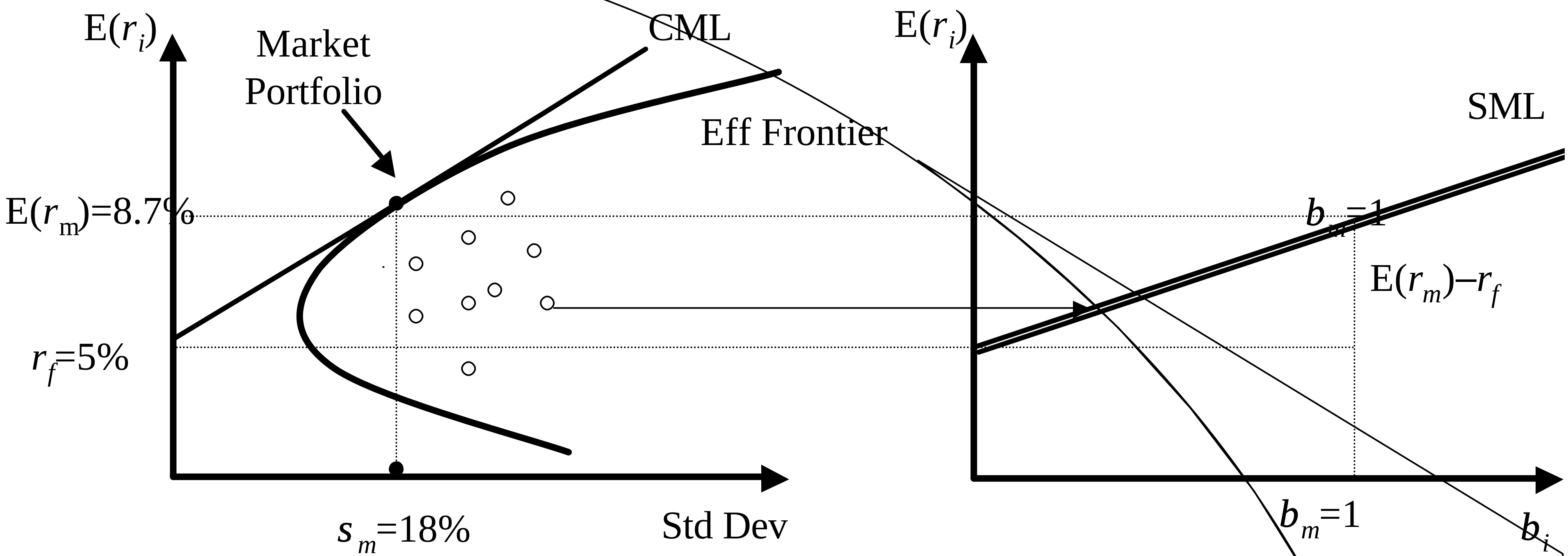
  - Implies nothing has better risk/variability ratio $[E(r) - r_f]/\sigma$ than market portfolio.



CAPM and Beta



- CAPM – Recall: total risk = systematic risk + non-systematic/diversifiable risk
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- Required (or expected) return via CAPM: $E(r_i) = r_f + b_i \times [E(r_M) - r_f]$
  - risk-free rate - pure time value of money ($ß_{rf} = 0$)
  - market risk premium - reward for bearing systematic risk ($ß_m = 1$)
  - beta coefficient - measure of systematic risk in asset *i*: $ß_i = \text{cov}(r_i, r_M)/\text{var}(r_M)$
  - Beta of portfolio is linear combination of asset betas. $b_p = a b_1 + (1-a) b_2$

CAPM and Beta's in Practice

- Betas – measure of systematic risk $\beta_i = cov(r_i, r_M)/var(r_M)$
  - http://moneycentral.msn.com/; www.reuters.com; finance.yahoo.com; MutualF.
  - Calculating beta: (1) get price data; (2) calculate returns (3) calculate covar/var.
- Betas in investment policy statements (IPS)
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- Betas in portfolio evaluation
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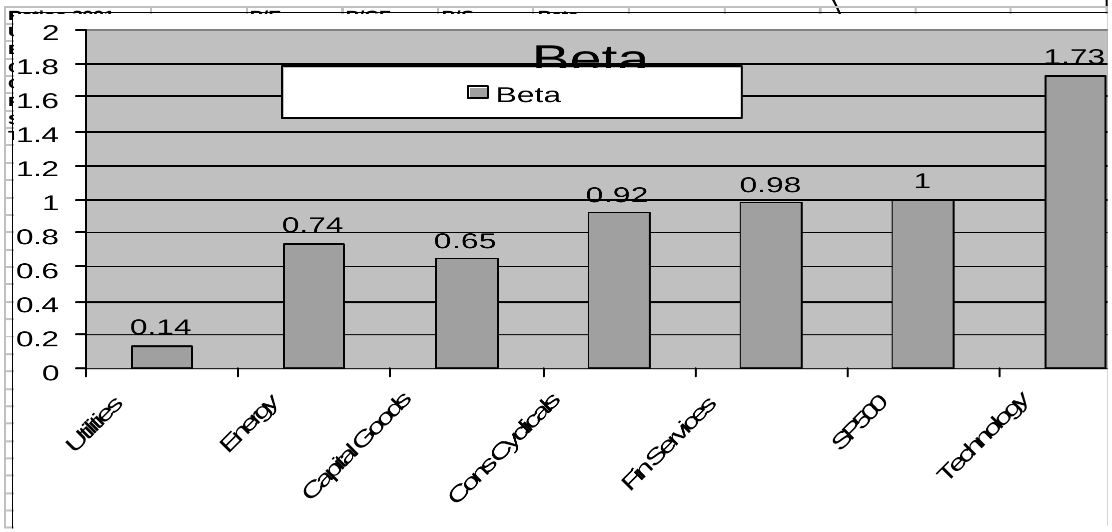
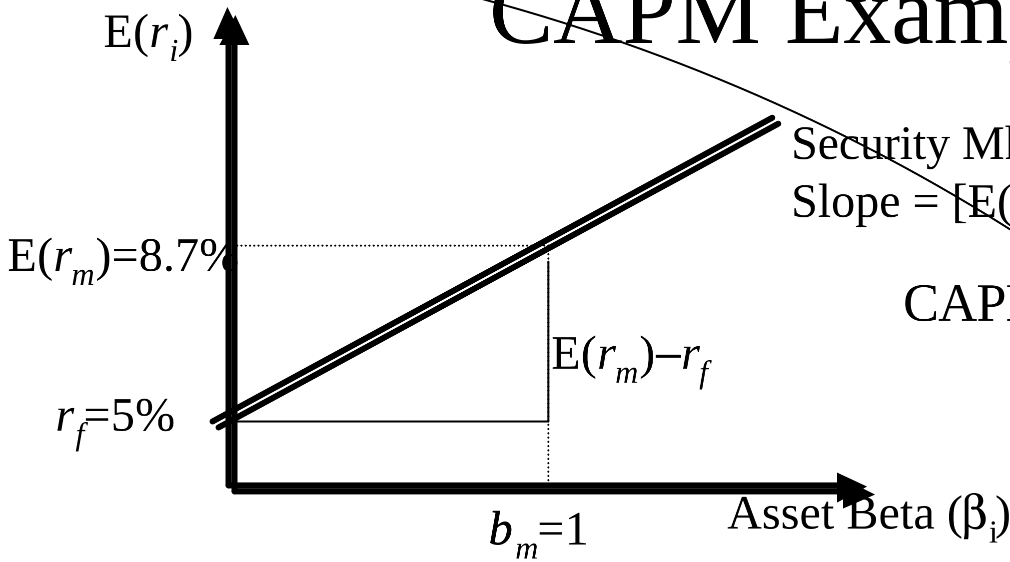
- Betas in corporate finance
  - –



CAPM Examples 1/2

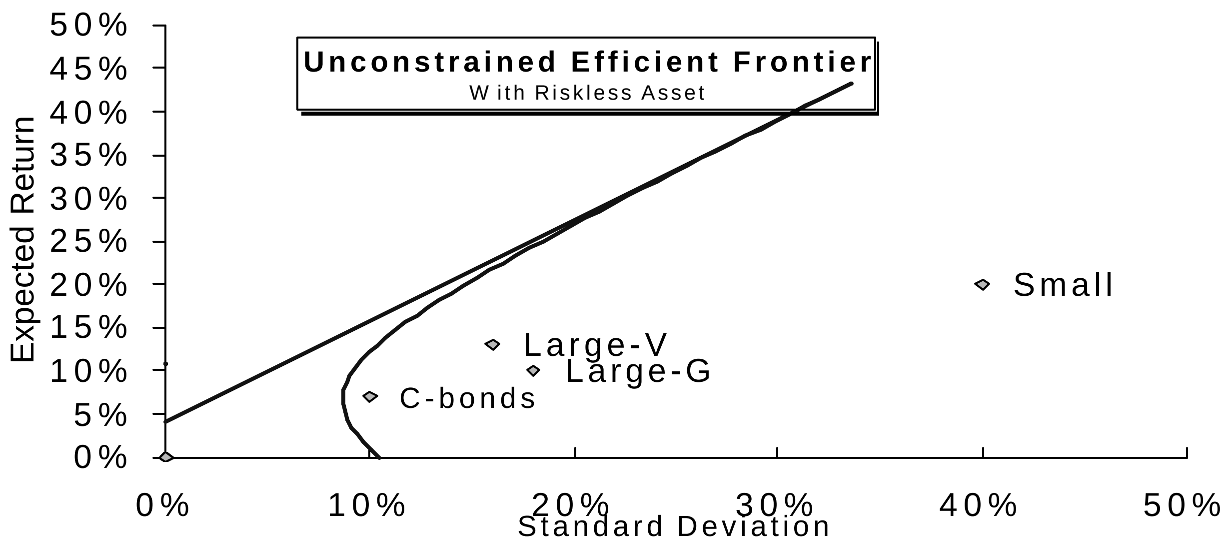


CAPM: $E(r_i) - r_f = b_i \times [E(r_M) - r_f]$

- **Beta and cost of capital:** Your company is considering a buyout of local coop whose principal line of business is generating and transmitting electricity. You estimate this business would generate FCF of $1 per share, with growth matching demand for electricity, ~3% annually. Prepare a valuation of this business. The market risk prem is 6%, risk-free rate is 5% and industry beta is ~0.14.
- A:
- A1:

- **Beta and pricing stocks:** The risk-free rate is 5% and the market risk prem is 6%. Assuming the CAPM holds, can stock have beta of 1.6 and expected return of 16%? A1:

CAPM Examples 2/2



- **Market portfolio:** The market portfolio has expected return of 12% and std dev of 16%. The risk-free rate is 4%. Can a portfolio have expected return of 10% and std dev of 10%?
- A:
- A1:
- A1:

- **Market portfolio:** The market portfolio has expected return of 12% and std dev of 16%. The risk-free rate is 4%. Can a portfolio have expected return of 10% and std dev of 20%?
- A:

CAPM: Tests of Understanding

- Under the CAPM, which asset is a riskier addition to a portfolio?

	$E(R_i)$	s_i
Chucky Cheese	11%	4%
Bison Burgers	5%	8%

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- If a stock high systematic risk, does that imply low expected return?
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- Suppose you can tolerate high risk, how might you construct a portfolio?
  - Should you invest it all in GOOG?
-
  - Recall total risk = systemic risk plus non-systematic/diversifiable risk.
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CAPM and Regression Models

- Recall CAPM: $E(r_i) - r_f = b_i \times [E(r_M) - r_f]$
  - One way to estimate beta is to use a regression!

- Consider the regression: $(r_i - r_f) = a_i + ß_i(r_m - r_f) + e_i$ *where*
  - $(r_m - r_f)$ = systematic (market) excess return (S&P 500).
    - $ß_i$ = exposure to systematic (market) risk
    - $ß_i(r_m - r_f)$ = portion of asset's return due to movements in market index.
    - e_i = unsystematic (idiosyncratic) return
    - a_i = expected return on asset if market excess return is zero.

- Regression model helps formally decomposes an assets' variance:

$$s_i^2 \quad = \quad b_i^2 \, s_m^2 \quad + s^2(e_i)$$

total variance =

.

- Example of regression for GM on next slide. To do this in Excel:

  Load Analysis Toolpack– Excel 07

  Button, Excel Options, Add-ins (Go), check Analysis Toolpack

- Run regression – Data, Data Analysis, Regression

CAPM and Regressions from Text

- Table 7.1 Monthly returns on T-bills, S&P 500 and GM, 1/99-12/03
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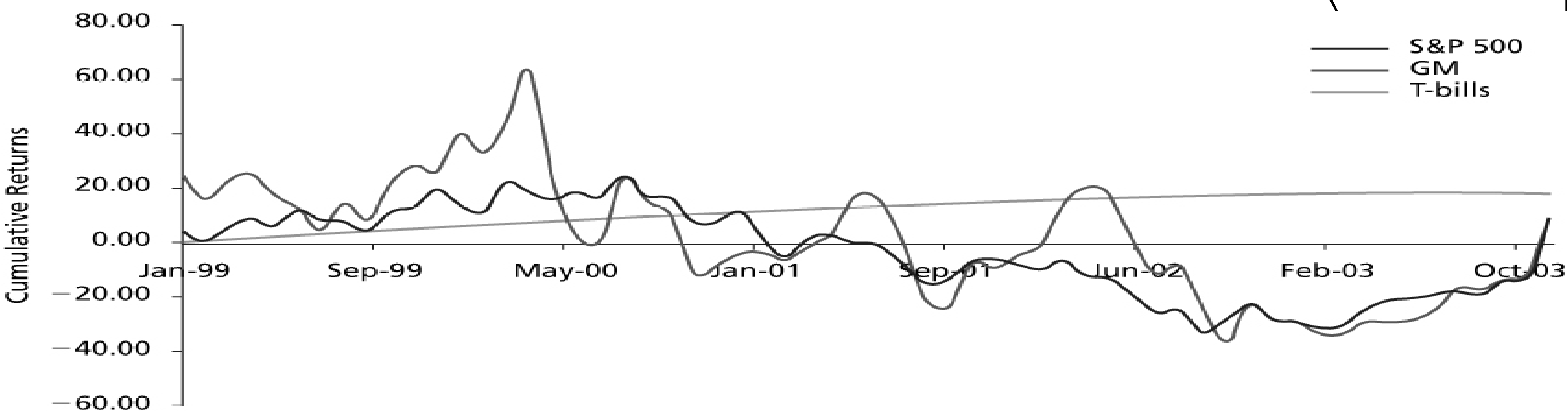
  – Note: authors used S&P 500 index, rather than total return.
  – Excess return on risk-free rate should be 0.

	T-Bills	S&P 500	GM
Average excess return (%)	0.28	−0.33	0.49
Standard deviation (%)	0.16	4.96	11.24
Geometric average (%)	0.28	−0.17	0.15
Cumulative total 5-year return (%)	18.20	−9.54	9.10

- Figure 7.3 Cumulative Returns on T-bills, SP500 and GM, 1/99-12/03



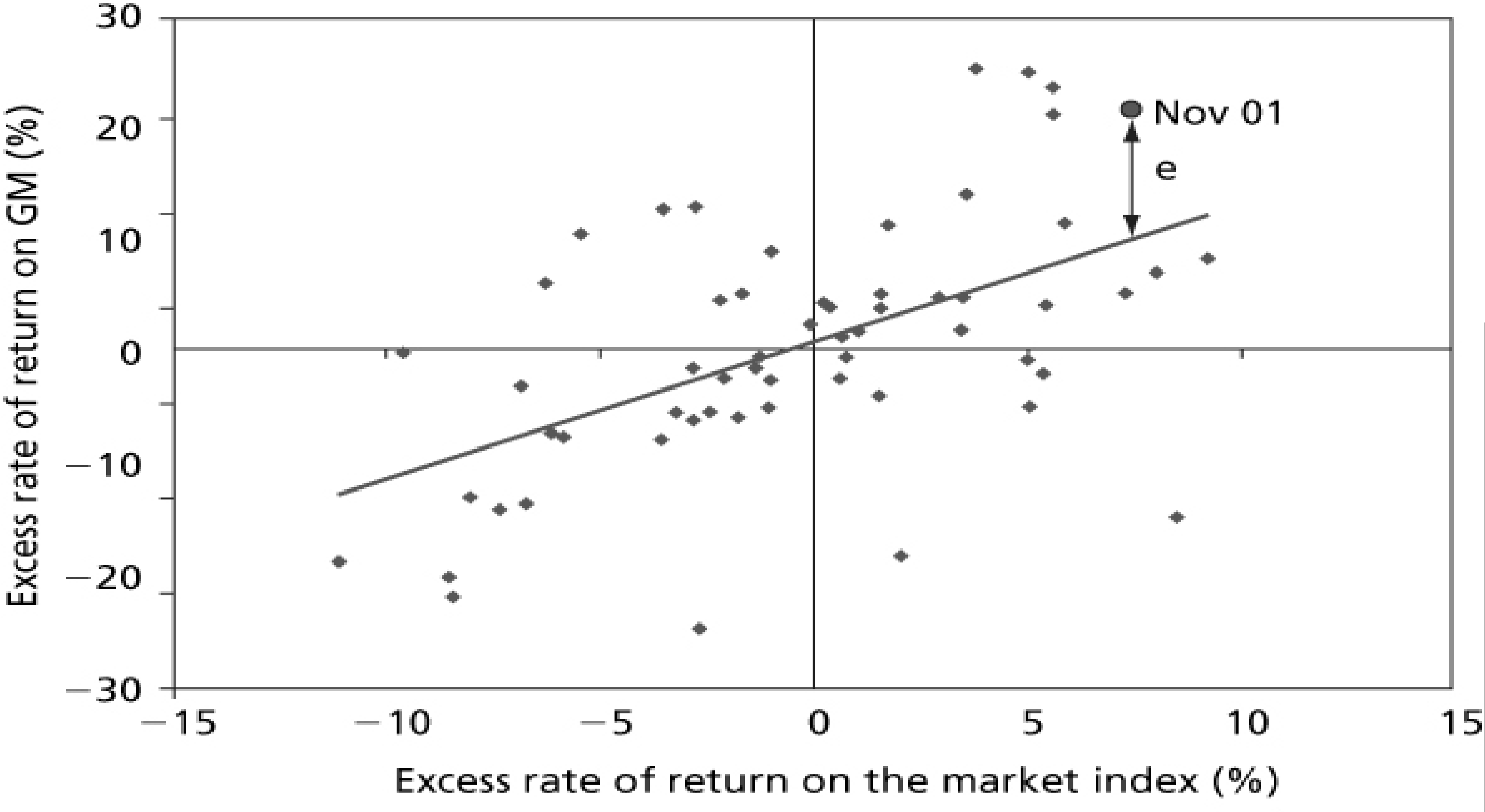
CAPM and Regressions from Text

- Table 7.2 – Regression of GM excess return on S&P 500 excess return.
  - CAPM: $E(r_i)–r_f = b_i \times [E(r_M)–r_f]$ Regression: $(r_i – r_f) = a_i + ß_i(r_M – r_f) + e_i$
- Conclusions (let XR = excess return):
  - Slope - Beta estimate is 1.24.
  - Adj R^2 -
  - Intercept (alpha) –
  - Systematic XR: $E(r_{GM})–r_f = b_{GM}[E(r_M)–r_f] = 1.24*–9.54 = –11.8\%$
    .
  - Unsystematic XR: Actual XR – systematic XR=9.1% – –11.8%=18.9%

Regression Statistics	
Multiple R	0.5465
R-square	0.2987
Adjusted R-square	0.2866
Standard error	9.4909
Observations	60

	Coefficients	Standard Error	t-statistic	p-value	Lower 95%	Upper 95%
Intercept	0.8890	1.2279	0.7240	0.4720	−1.5690	3.3470
Slope	1.2384	0.2492	4.9697	0.0000	0.7396	1.7372

CAPM and Regression from Text



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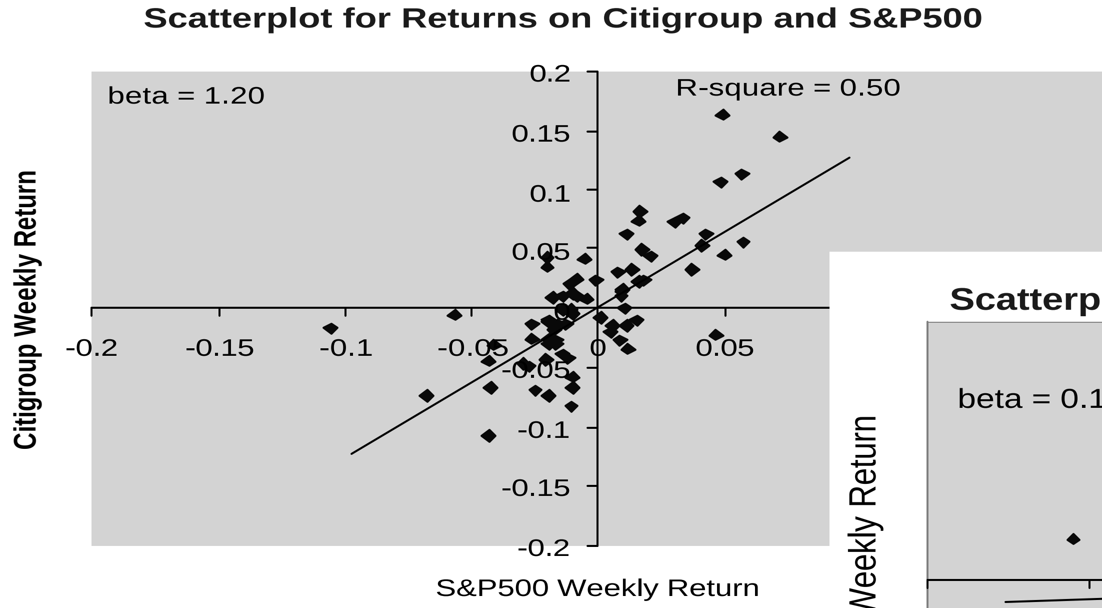
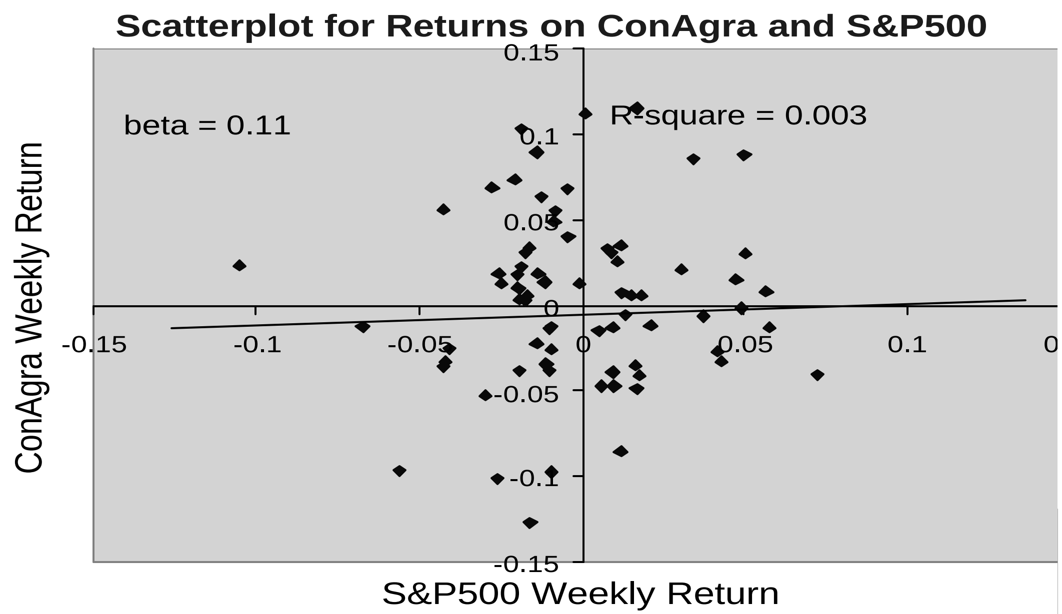
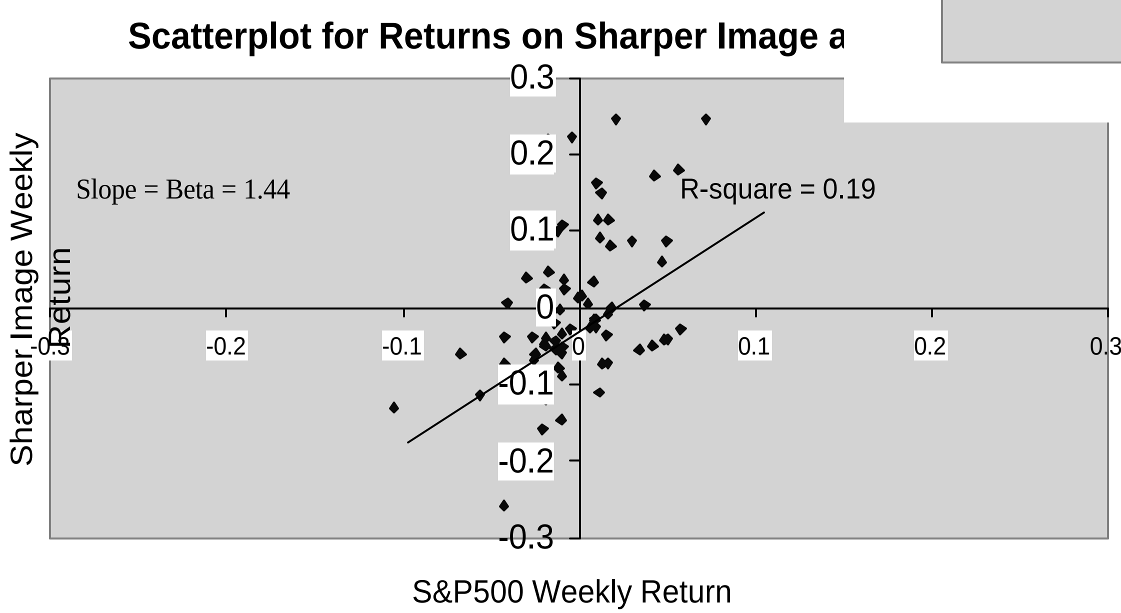


Scatterplot for Returns on Citigroup and S&P500
beta = 1.20
R-square = 0.50
Citigroup Weekly Return
S&P500 Weekly Return
0.2
0.15
0.1
0.05
-0.05
-0.1
-0.15
-0.2
-0.2
-0.15
-0.1
-0.05
0
0.05



Scatterplot for Returns on ConAgra and S&P500
beta = 0.11
R-square = 0.003
ConAgra Weekly Return
S&P500 Weekly Return
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.15
-0.1
-0.05
0
0.05
0.1
0



Scatterplot for Returns on Sharper Image a
Slope = Beta = 1.44
R-square = 0.19
Sharper Image Weekly Return
S&P500 Weekly Return
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.3
-0.2
-0.1
0
0.1
0.2
0.3

Multifactor Models

- CAPM – relies on one factor, the market, to explain systematic stock returns.
  - Empirically other factors help explain why some stocks earn higher returns.
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- Fama-French three factor model: $r_i – r_{f,} = a_i + b_M(r_N – r_{f,}) + b_{HML} r_{HML} + b_{SMB} r_{SMB} + e_i$
  - Market index – return on market minus risk-free rate.
  - Size (SMB) – return to small firms minus big firms.
  - Book-Market ratio (HML) – return to firms with high B/M minus low B/M.
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table 7.4
Regression statistics for the single-index and the FF three-factor model

	Single-Index Regression (broad market index)	FF Three-Fa Model
Correlation coefficient	0.54	0.60
Adjusted R-square	0.27	0.32
Regression standard error	9.57	9.24
Intercept	0.60	0.30
Standard error	1.24	1.24
Market beta	1.16	1.26
Standard error	0.24	0.24
SMB beta	—	0.05
Standard error	—	0.29
HML beta	—	0.52
Standard error	—	0.22

Tips for the Savvy Investor from CAPM and Multifactor models

- CAPM yields simple statistic for summarizing the risk of a portfolio (beta).
  - Optimal (tangent) portfolio is the market portfolio.
  - No portfolio has higher return-to-variability ratio than the market portfolio.
  - Capital Market Line is the name for the "best" CAL with slope=[E(r_m)–r_f]/ s_m.
  - Expected returns on all assets satisfy CAPM: $E(r_i) - r_f = b_i \times [E(r_M) - r_f]$.
  - Security market line describes CAPM equation Slope=[E(r_i)–r_f]/b_i =[E(r_m)–r_f].

- Total risk depends on both systematic and unsystematic/diversifiable risk.
  - Total risk (s_i^2) = systematic risk ($b_i^2 s_m^2$) + unsystematic/diversifiable risk (s^2_{ei})
  - Construct portfolios with little (or no) unsystematic/diversifiable risk!

- Mutlifactor models – It is difficult, but not impossible, to find factors other than "market" that contribute to explaining variation in returns.
  - The market explains

- APT and multifactor models introduce additional methods for reducing the dimensionality of the portfolio mgt problem (see www.birr.com).
  - APT risk factors explain