

6. Risk and Diversification

- Risk
 - Example
- Diversification
 - Systematic vs Unsystematic Risk
- Portfolio Theory
 - Efficient Frontier with two assets,
 - Efficient Frontier with many assets
 - Efficient Frontier with many assets plus riskless asset
- Applications of Portfolio Theory
 - Asset Allocation
 - Risk Assessment

Statistics Review

- Statistics review– For random variable with given distribution (\sum refers to $\sum_{s=1}^S$):
 - Let $p(s)$ be the probability of state s and $r(s)$ be the return if state s happens.
 - Mean: $E(r) = \sum p(s)r(s)$
 - Variance: $\text{Var}(r) = \sigma^2 = \sum p(s) [r(s)-E(r)]^2$ and $\text{SD}(r) = \sigma = \text{sqrt}(\sigma^2)$
 - Covariance: $\text{Cov}(r_i, r_j) = \sum p(s) [r_i(s)-E(r_i)][r_j(s)-E(r_j)]$
 - Correlation: $\rho = \text{Cov}(r_i, r_j) / \sigma_i \sigma_j$
 - Note 1: for historical sample, use $p(s)=1/S$ or, for variance, $p(s)=1/(S-1)$
 - Note 2: for historical sample, can use Excel functions =average, =var, =stdev

- Statistics review – combining assets (random variables) into portfolio:
 - Let w be the fraction of your portfolio invested in asset i .
 - Mean of portfolio: $E(r_p) = wE(r_i) + (1-w)E(r_j)$
 - Variance of portfolio: $\text{Var}(r_p) = \sigma_p^2 = w^2 \sigma_i^2 + (1-w)^2 \sigma_j^2 + 2w(1-w)\rho \sigma_i \sigma_j$

Risk: Example

- Consider the return distributions on the following two stocks.
 - One is cyclical, the other counter-cyclical.

Economy	$p(s)$	$r_i(s)$ ANF	$r_i(s)$ WMT
Strong	0.25	20%	5%
Normal	0.50	10%	10%
Recession	0.25	0%	15%
	$E(r)$	10.00%	10.00%
	$\text{Var}(r)$	0.0050	0.0013
	$\text{Std}(r)$	7.07%	3.54%

- **You have \$300 million to invest. What should you do?**

Risk: Example

- Consider investing \$100 in ANF and \$200 in WMT.

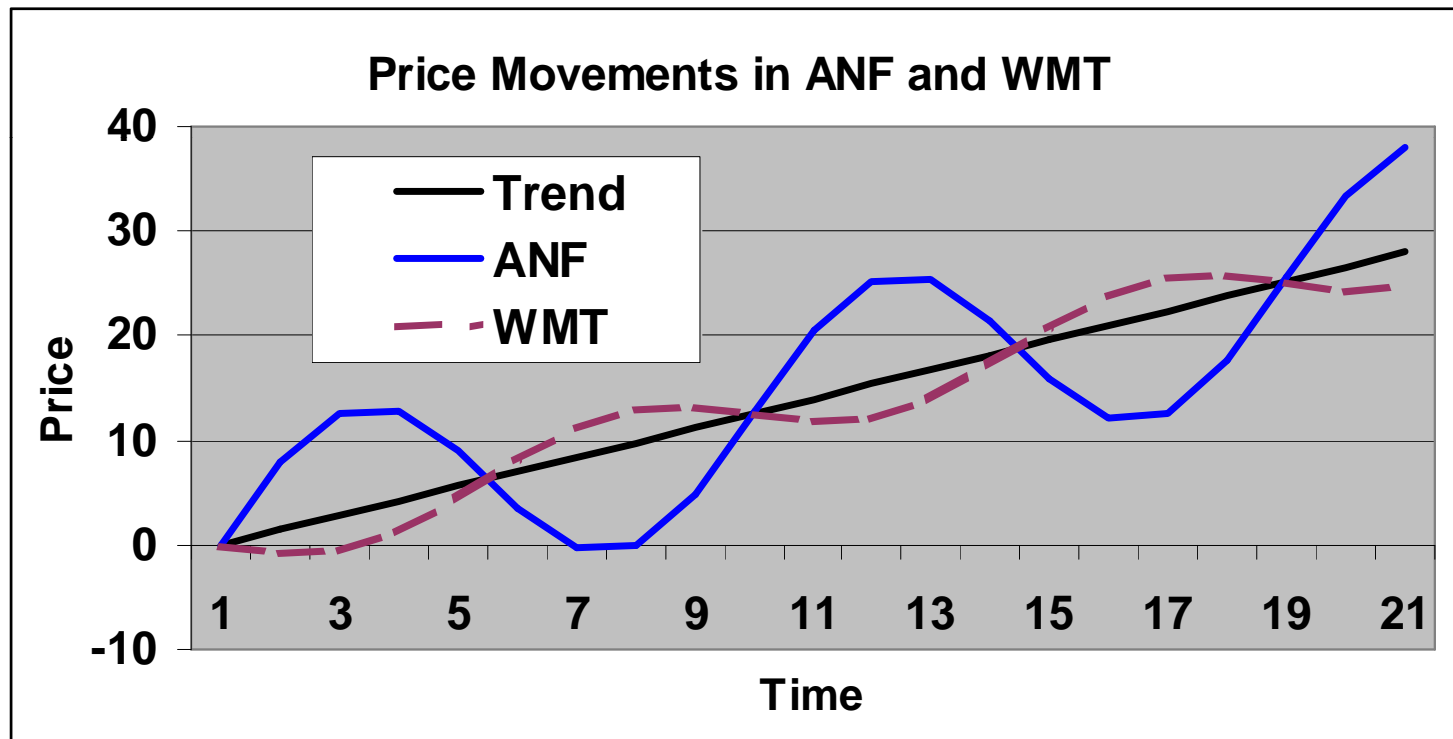
		Gross Return			% Return
Economy	Prob	\$100 in ANF	\$200 WMT	on Portfolio	on Portfolio
Strong	0.25	120	210	330	10.00%
Normal	0.50	110	220	330	10.00%
Recession	0.25	100	230	330	10.00%
	E(r)	10.00%	10.00%		10.00%
	Std	7.07%	3.54%		0.00%



- Verify at home: with $w = 1/3$, $\rho = -1$, $\sigma_i = 7.07$; $\sigma_j = 3.54$
- $E(r_p) = wE(r_i) + (1-w)E(r_j)$
= **10%**.
- $\text{Var}(r_p) = w^2\sigma_i^2 + (1-w)^2\sigma_j^2 + 2w(1-w)\rho\sigma_i\sigma_j$
= **0%**
- When $\rho = -1$, it is possible to form a perfect hedge!

Risk: How did that happen?

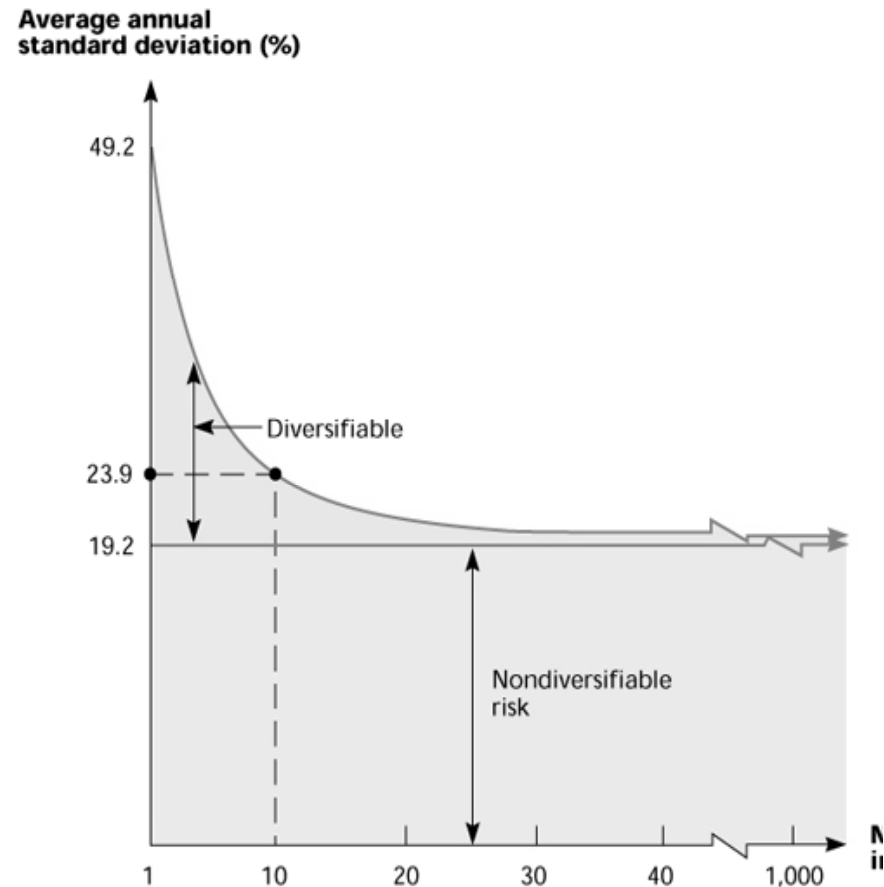
- How did we create a portfolio with no volatility?
 - When the correlation between two assets is -1 ,



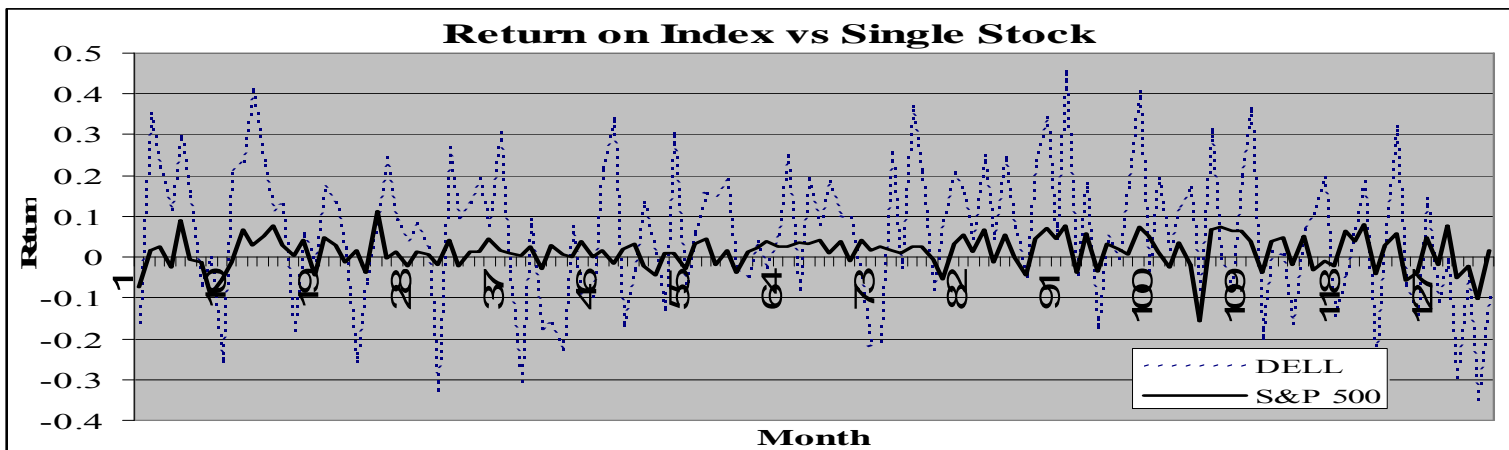
Risk: Diversification 1/2

- **Total Portfolio risk (volatility) = non-diversifiable risk + diversifiable risk**
 - .
- Do you want to lower the volatility of your portfolio?
 - Just add more stocks until diversifiable risk is (almost) gone!
 -

# Stocks	Avg SD	Ratio of Portfolio SD to
1	49.24%	1.00
2	37.36%	0.76
4	29.69%	0.60
6	26.64%	0.54
8	24.88%	0.51
10	23.93%	0.49
20	21.68%	0.44
30	20.87%	0.42
40	20.46%	0.42
50	20.20%	0.41
100	19.69%	0.40
200	19.42%	0.39
300	19.34%	0.39
400	19.29%	0.39
500	19.27%	0.39
1000	19.21%	0.39



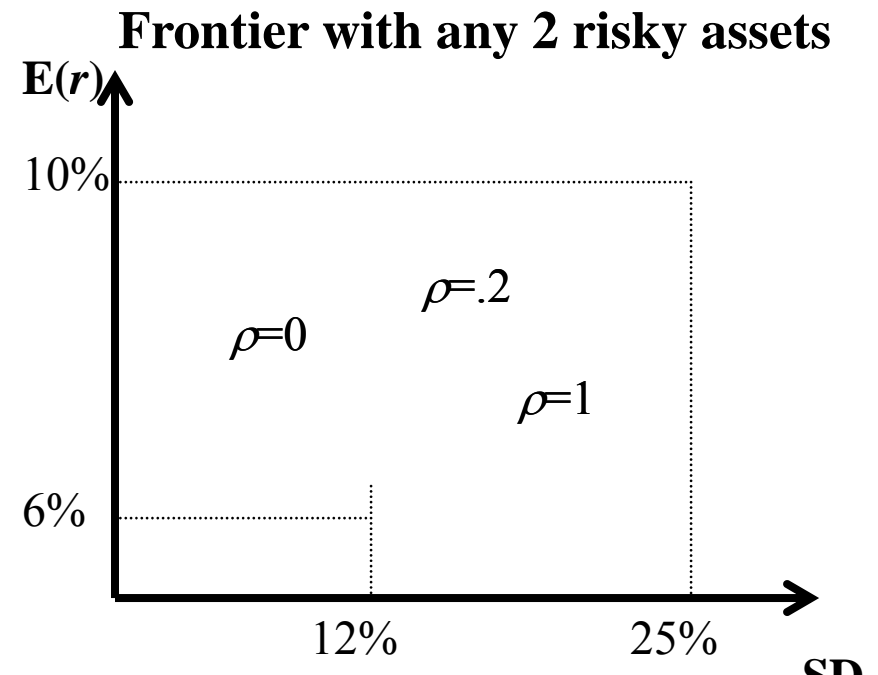
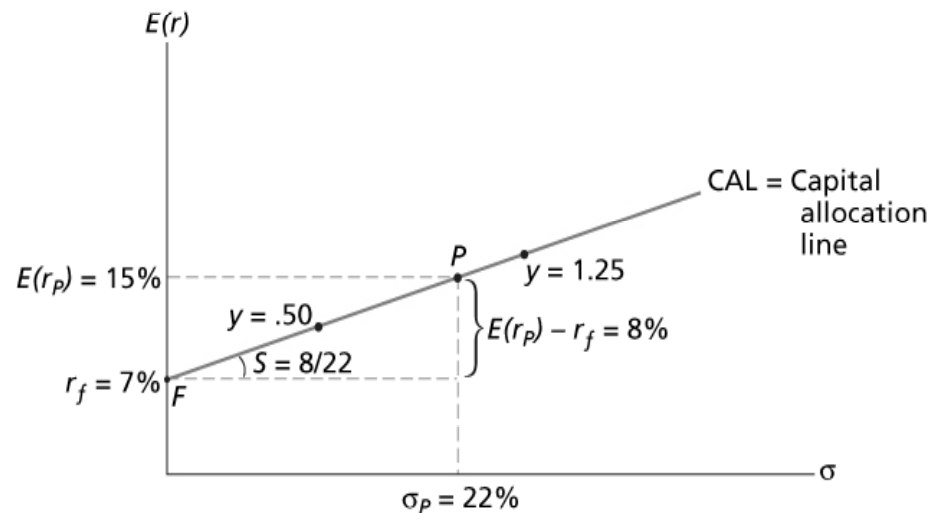
Risk: Diversification 2/2



- **Total Portfolio risk (volatility) = non-diversifiable risk + diversifiable risk**
- Non-diversifiable Risk (aka market or systematic risk).
 -
- Diversifiable Risk (aka firm-specific or idiosyncratic or unique risk).
 - risk of industry declining; company performing poorly; CEO dies in car crash; demand for product being low.
 -
- Large investors do not require compensation to bear diversifiable risk.
 - They can hold large portfolios and so make diversifiable risk go away.
 -

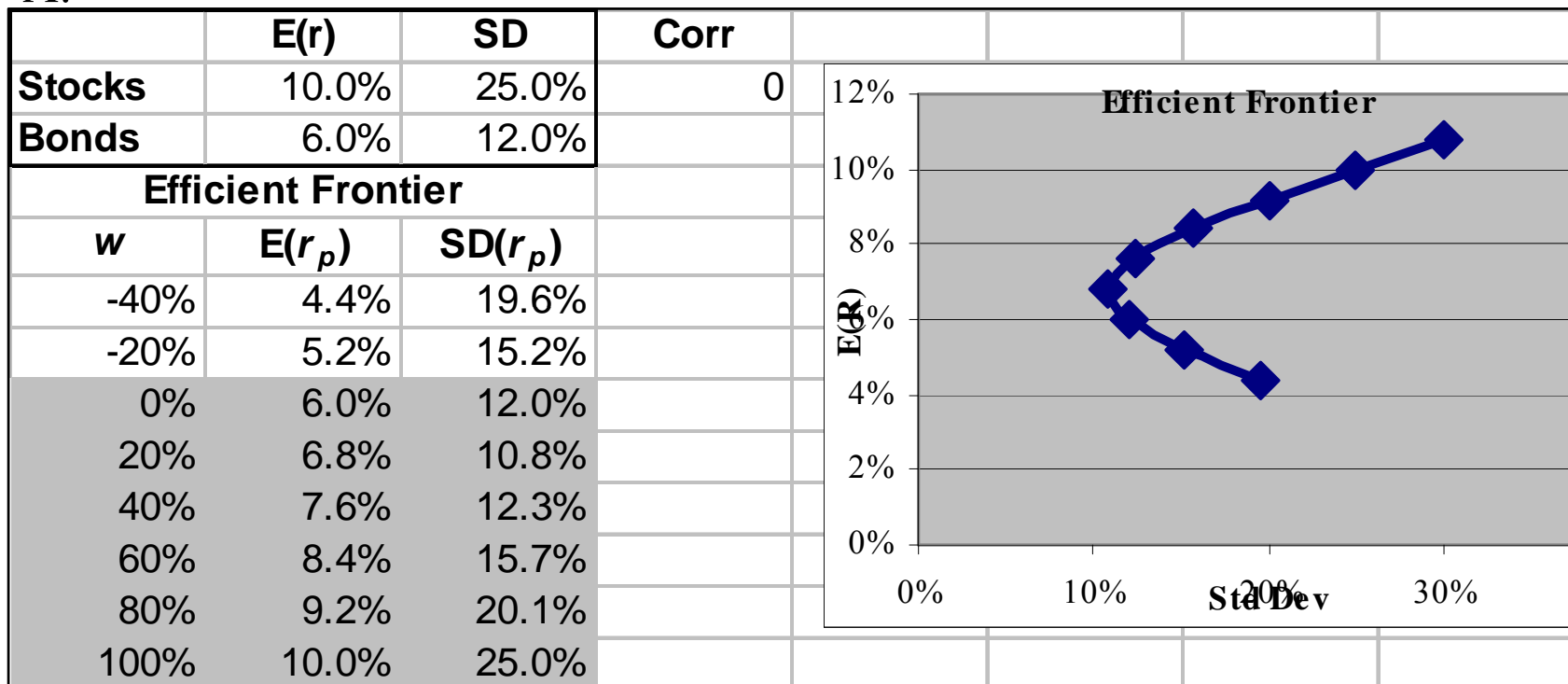
Efficient Frontier: Two Risky Assets

- What does Efficient Frontier with two risky assets look like?
 - The Efficient Frontier tells us expected return and std dev on portfolios.
 - $E(r_p) = wE(r_i) + (1-w)E(r_j)$
 - $\text{Var}(r_p) = w^2\sigma_i^2 + (1-w)^2\sigma_j^2 + 2w(1-w)\rho\sigma_i\sigma_j$
- Old frontier with one risky asset ($\sigma > 0$) and T-bills ($\sigma = 0$)
- New frontier with two risky assets ($\sigma > 0$) is different.
 - In general, it is hyperbola (if $-1 < \rho < 1$)!



Efficient Frontier: Two Risky Assets

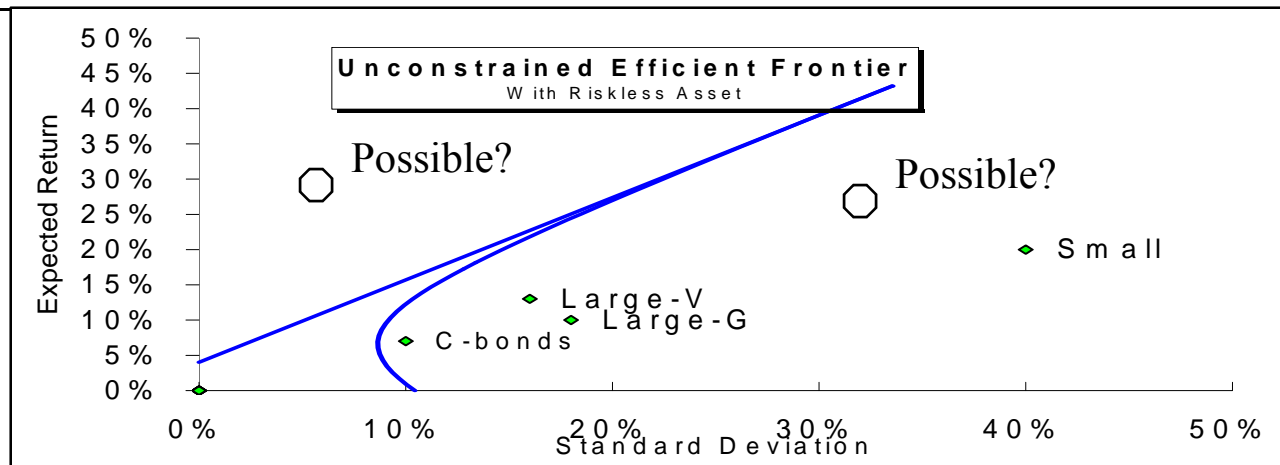
- Efficient Frontiers with two assets –
 - See [Harvey's Java Tool](#) and book web page.
 - Min Variance portfolio
- Q: Suppose you are risk averse. How should you allocate your portfolio between stock and bond funds?
- A:
- Q: Would you ever choose an allocation on lower half of frontier?
- A:



Efficient Frontier: Is this for Real? 1/2

- Do investors really use Efficient Frontiers?
 - Yes, portfolio mgrs use “optimization” software that allows many investments.
 - Typically applied to asset classes (stocks by size and growth, bonds) or sectors.
 -
- What does the efficient frontier with many risk assets look like?
 - It is a hyperbola, just like with two assets, although indiv assets on interior.

	E(R)	σ	Large V	Large G	Small	Corp Bond
Large-Value	13%	16%	1	0.90	0.70	0.10
Large-Growth	10%	18%		1	0.90	0.10
Small Cap	20%	40%			1	-0.15
Corp Bond	7%	10%				1



Efficient Frontier: Test of Understanding 1 ¹¹

- Q: You manage funds for wealthy indiv and may add one stock to existing assets.
 - Both firms are stable and financially sound and market conditions will persist.
CMSV: $E(r_1)=10\%$ and $\sigma_1=5\%$ NVDM: $E(r_2)=20\%$ and $\sigma_2=4\%$.
Which asset to you add?
- A:

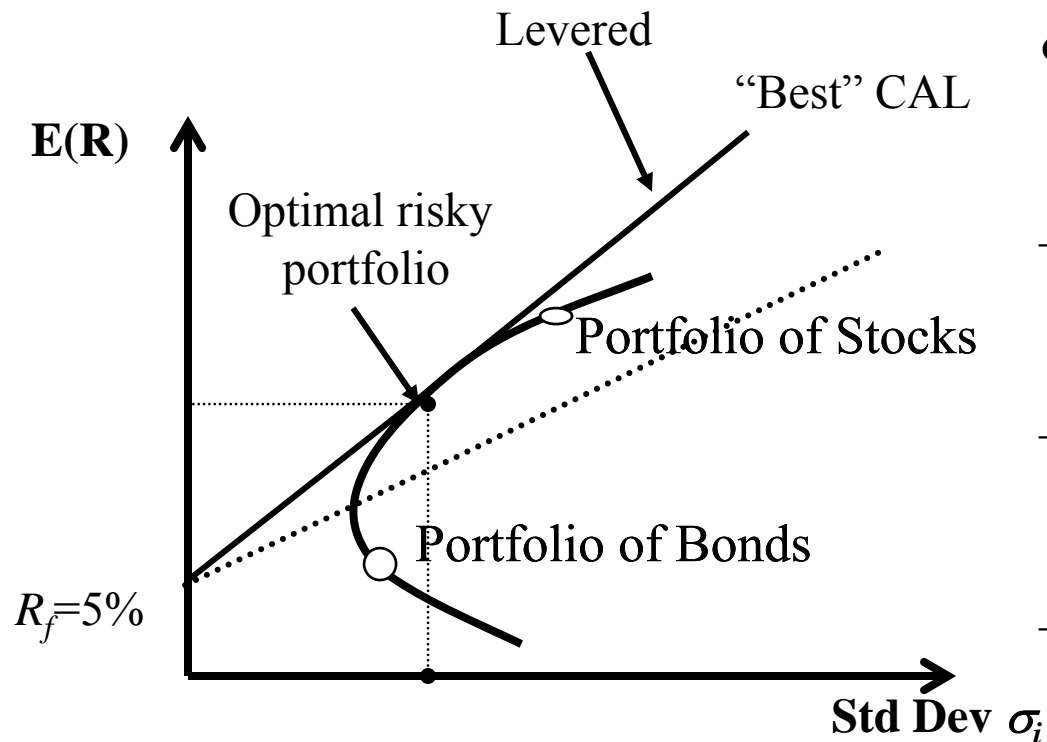
- Q: Commodity investments have a lower expected return than stocks and low correlation with stocks. Should you add them to your portfolio?
- A:

- Q: Are there other real applications of this kind of variance analysis?
- A2:
 - Near retirement, less time to alter savings behavior to make up for losses.
- A1: Yes.
 - Estimates probability portfolio will drop 10% today. Used by all major banks.
 - Popular after crisis in Orange County, Barings PLC (2000) led to bankruptcies.

Efficient Frontier: Test of Understanding 1/2 ¹²

- **Suppose we calculate an efficient frontier over a 5-yr period (2003-2008).**
- Q: Suppose portfolio mgr in 2008 with fixed portfolio has performed below frontier. What does that mean?
- A:
 -
- Q: Can a portfolio manager beat the efficient frontier? (i.e., be outside hyperbola?)
- A1:
- A2:
 -
 - Beating efficient frontier requires an active strategy (later)!

Efficient Frontier: Risky and Riskless Asset



- Suppose now that investors have access to risky assets plus T-bills.
 - How does this alter the investment opportunities?
 - We know that risky fund plus T-bills produces a CAL.
 - **Which CAL is best?**

- Best Capital Allocation Line (CAL) –
- How might you allocate your portfolio between stocks, bonds, T-bills?
 - (1) Identify optimal risky portfolio based on expected returns/variances.
 - (2) Mix optimal risky portfolio with riskless asset based on preferences for risk.
 - If investors have same beliefs on expected returns and var, all should hold same optimal portfolio of risky assets. Optimal portfolio may be broad index fund.

Strategy for Active Portfolio Mgt

- Index portfolio to benchmark, such as SP500, based on risk tolerance
- Strategic Allocation – of asset class with 5-10 yr horizon
 - SP500, Bonds, Int'l investments
 -
- Tactical Allocation – of asset class (stocks, bonds, bills) with 1-12 mo horizon
 - Make tactical bet to over- or under-weight asset class
 - Based on current, arriving info, such as current macro/market analysis.
- Sector rotation – within asset class
 - Bond sectors may be Govt, Corp, Mortgage, Long, Short.
 - Stock sectors may be economic (8-10) for based on capitalization/growth
 -
- Asset selection – determines 10%-20% of portfolio variation.
 - Which individual securities are under-priced?
 - 20% of variation determined by individual asset selection.
 -

Tips for the Savvy Investor from Risk and Diversification

- Risky Assets (with correlation < 1) can be used as a hedge against one another.
 - Mean of portfolio: $E(r_p) = wE(r_i) + (1-w)E(r_j)$
 - Variance of portfolio: $\text{Var}(r_p) = \sigma_p^2 = w^2\sigma_i^2 + (1-w)^2\sigma_j^2 + 2w(1-w)\rho\sigma_i\sigma_j$
 - Frontier with two or more risky assets (both have $\sigma > 0$) is hyperbola (if $|\rho| \neq 1$)!
 - Frontier with one risky asset ($\sigma > 0$) and riskless asset ($\sigma = 0$) is a line! (CAL)
- A portfolio of assets will have (much) less volatility than the individual assets.
 - Total risk = systematic risk plus non-systematic risk.
 - Diversified portfolios remove all non-systematic/idiosyncratic risk.
- For an individual asset, volatility is not the relevant measure of risk.
 - Appropriate risk measure should be related to correlation or covariance.
 - For a diversified portfolio, volatility is an appropriate measure of risk.
- Portfolio theory is used to formulate “asset allocation” strategies.
 - Also provides theoretical foundation for “passive” investing strategies.