CHAPTER 6

THE STRUCTURE OF INTEREST RATES

Outline: Factors Influencing Interest Rate Differences

- Term to Maturity –
  - Relationship between yield and term to maturity on securities that differ only in length of time to maturity.
  - Notation:
    - Today's 1-year spot rate is ${}_tR_1$. Today's 2-year spot is ${}_tR_2$
    - The one-year *forward* rate for beginning in one year (yr 2) is ${}_{t+1}f_1$
  - **Pure Arbitrage** – ability to earn return with no net investment and no risk.

- Default Risk: Default Risk Premium = $i - i_{rf}$

- Tax Treatment – After-tax return, i_{at}, is found by multiplying the pre-tax return by one minus the investor's marginal tax rate: $i_{at} = i_{bt}(1-t)$

- Marketability

- Options on Debt Securities: Call, Put or Convertibility option
  - *call interest premium* CIP = $i_c - i_{nc}$
  - *put interest discount* PID = $i_p - i_{np}$
  - *conversion yield discount* (CYD) CYD = $i_{con} - i_{ncon}$.

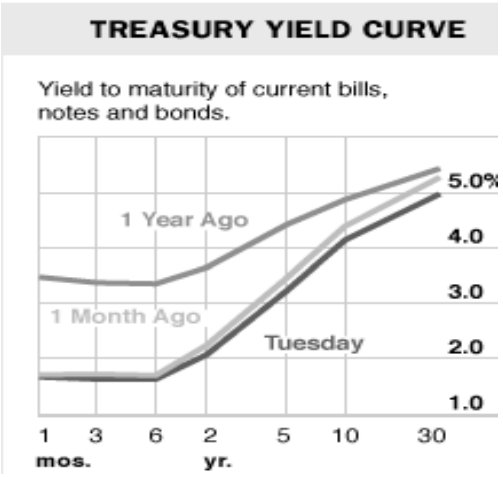
Term to Maturity and Interest Rates

- Yield curve - graph of yields versus time to maturity, usually for Treasury bonds. Usually slopes up, as long-term rates are usually greater than short-term.
  - 3 possible explanations for yield curve (See FRBSL; nasdbondinfo.com)

- (1) Market Segmentation theory– Short- and long-term bonds trade in distinct markets, and rates are determined independently.
  - (1b) Preferred habitat theory

- (2) Liquidity Preference theory - investors require risk premium to induce them to hold long-term bonds, since most investors prefer short horizons.

- (3) Expectations theory– observed long-term rate is an average of today's short-term rate and expected future short-term rates (which are foreward rates)..



Interest Rates -

	Zero #1			Zero #2	Bond "X"
Coupon	**0%**			**0%**	**20% Annually**
Maturity	**1 year**			**2 year**	**2 year**
Par	***100***			***100***	**1000**
Price	**92.59**			**84.18**	***???***
YTM (BEY)	**8%**			**8.995%**	
Time	**0**	**1**	**2**	**3**	**…** **20**
(Z1) CF	**92.59**	100			
(Z2) CF	**84.18**		100		
(BX) CF	**???**	200	200+ 1000		

Suppose we know the price of two zero coupon bonds.
Can we find the price of the coupon bond "X" using the price of the zeros??

"no arbitrage" principles suggest:

PV(X) =

"Spot" Interest Rates

- In general, bonds with same maturity but different coupons have different YTMs
  - YTM on 2 yr coupon (8.91%) differed from YTM on zero (8.995%).
  - Arbitrage forces such interest rates to be linked.
    - bonds must have same default risk, tax status, no embedded options, etc..

- Spot Rates – the YTM on zero coupon (Treasury) bonds
  - E.g., the 1-yr spot was 8%; 2-yr spot was 8.995%
  -

- Forward rates – inferred future (short-term) interest rate that would equate the YTM on a long-term bond with strategy of rolling over shorter term bonds.
  - Given sequence of spot rates, we can find forward rates.
  -

Time	**0**	**1**	**2**
1 year zero		**-8% spot-**	
2 year zero		**-8% spot-**	**--- ????--- forward?**
2 year zero		**----------8.995% spot----------**	

Forward Interest Rates

Time	**0**	**1**	**2**
1-year zero		**-8% spot ($_tR_1$)-**	
2-year zero		**-8% spot ($_tR_1$)-**	**- forward ($_{t+1}f_1$)?**
2-year zero		**----------8.995% spot ($_tR_2$)--------**	

•We observe today's **1-year spot ($_tR_1$=8%)** and today's **2-year spot ($_tR_2$=8.9995%).**

•What is implied one-year ***forward*** **rate for year 2 ($_{t+1}f_1$)** ?
(This equates the return on a 2-year zero with two sequential 1-yr zeros)

• $(1+{}_tR_2)^2 = (1+{}_tR_1)(1+{}_{t+1}f_1)$
$(1+.08995)^2 = (1.08)(1+{}_{t+1}f_1)$

- The forward rate for *n* years ahead ($_{t+n-1}f_1$) is derived from spot rates $_tR_n$ and $_tR_{n-1}$

$$(1+{}_tR_n)^n = (1+{}_tR_{n-1})^{n-1}(1+{}_{t+n-1}f_1)$$

$$(1+{}_{t+n-1}f_1) = \frac{(1+{}_tR_n)^n}{(1+{}_tR_{n-1})^{n-1}}$$

Forward Rates

T	Price of Zero	Spot $_1R_n$	Forward $_nf_1$	Price from spot rates	Price from forward rates
1	$925.93	8.00%		1,000/1.08 = $925.93	
2	841.75	8.995	*10%*	$1{,}000/(1.08995^2)=841.85$	1000/(1.08)(1.1)

- Forward Rates – implies that earning 8.995% over 2 years is the same as earning 8% on the first year and 10% on the second.

- That is, by definition of forward rate
  - $(1+.08995)^2 = (1.08)(1+{}_{t+2}f_1)$
  - $(1+.08995)^2 = (1.08)(1.10)$

- So, to find price of, say, a 2-year bond today , you could either
  - $1{,}000/(1+.08995)^2 =$ (discount twice at todays 2-yr spot rate)
  - 1,000/(1.08)(1.10) = (discount once at today's spot rate and once at current forward rate for next year.)

More on Expectations and Liquidity....

- Consider the following two investments:
  (1) Buy 2 year zero. Earn average of 8.995%; same as 8% then 10%.
  (2) Buy 1 year zero, then another. Earn 8%, then $E({}_2r_1)$.

- Expectations hypothesis says that if investors don't care about liquidity, then the expected return on (1) must equal the return on (2):

$(1+{}_tR_2)^2 \equiv (1+{}_tR_1)(1+{}_{t+1}f_1)$ must equal $(1+{}_tR_1)(1+E({}_{t+1}R_1))$

${}_{t+1}f_1$ must equal $E({}_tR_2)$

next years forward rate must equal what markets expect to prevail.

- Liquidity preference theory says that investors prefer liquidity. Then strategy (1) is more risky, and must offer risk premium, so its expected return exceeds (2):

$(1+{}_1R_2)^2 \equiv (1+{}_1R_1)(1+{}_2f_1)$ is greater than $(1+{}_1R_1)(1+E({}_2R_1))$

${}_2f_1$ is greater than $E({}_2R_1)$

${}_2f_1$ = $E({}_2r_1)$ + liquidity premium

More on theories of the yield curve

- (1) Market Segmentation theory– Short- and long-term bonds trade in distinct markets, and rates are determined independently.
  - Implies that discontinuities and spikes are possible in the yield curve.

- (1a) Preferred Habitat theory - extension of the Market Segmentation.
  - investors trade outside of preferred maturity if adequately compensated.
  - .

- (2) Liquidity Preference theory - investors require risk premium to induce them to hold long-term bonds.
  - .

- (3) Expectations theory– observed long-term rate is an (geometric) average of today's short-term rate and forward rates (expected future short-term rates.)
  - Implies shape of the yield curve is determined solely by expectations of future interest rate movements.
  - **Upward sloping yield curve**: future interest rates are expected to ***increase***.
  - **Inverted yield curve**:

Yield Curve and Monetary Policy

- Q: Suppose the expectations theory holds. A 1-yr bond yields 2% today, and is expected to yield 4% next year. What should be the yield on 2-yr bond?
  A: About 3%.

- Q: What factors determine next year's expected rates?
  A:

- Fed affects short-term (ST) interest rates directly, but not long-term (LT) rates.
  - Policy affects long-term rates primarily through expectations hypothesis

- Q1: What happens if Fed temporarily lowers funds rate below LT average?
- A1:

- Q2: What happens if Fed persistently lowers funds rate below LT average?
- A2:

- Q3: How can the Fed keep long-term rates as low as possible?
- A3:

Example on the Term Structure (1)

T	Price of Zero	Spot ${}_1R_n$	Forward ${}_nf_1$
1	$925.93	8.00%	*8%*
2	841.75	8.995	*10%*
3	758.33	9.660	*11%*
4	683.18	9.993	*11%*

- **Q:** Prove that the forward rate for years 2 and 3 equals 11%.
- **A:** $(1+{}_tR_n)^n = (1+{}_tR_{n-1})^{n-1}(1+{}_{t+n-1}f_1)$

- **Q:** Suppose expectations hypothesis holds. Find next year's expected 1-yr spot rate.
- **A:**

Example on the Term Structure (2)

T	Price of Zero	Spot ${}_1R_n$	Forward ${}_nf_1$
1	$925.93	8.00%	*8%*
2	841.75	8.995	*10%*
3	758.33	9.660	*11%*
4	683.18	9.993	*11%*

• **Q:** Are the observed forward rates consistent with expectations hypothesis or the liquidity preference hypothesis?

•**A:**

Which Theory best describes term structure?

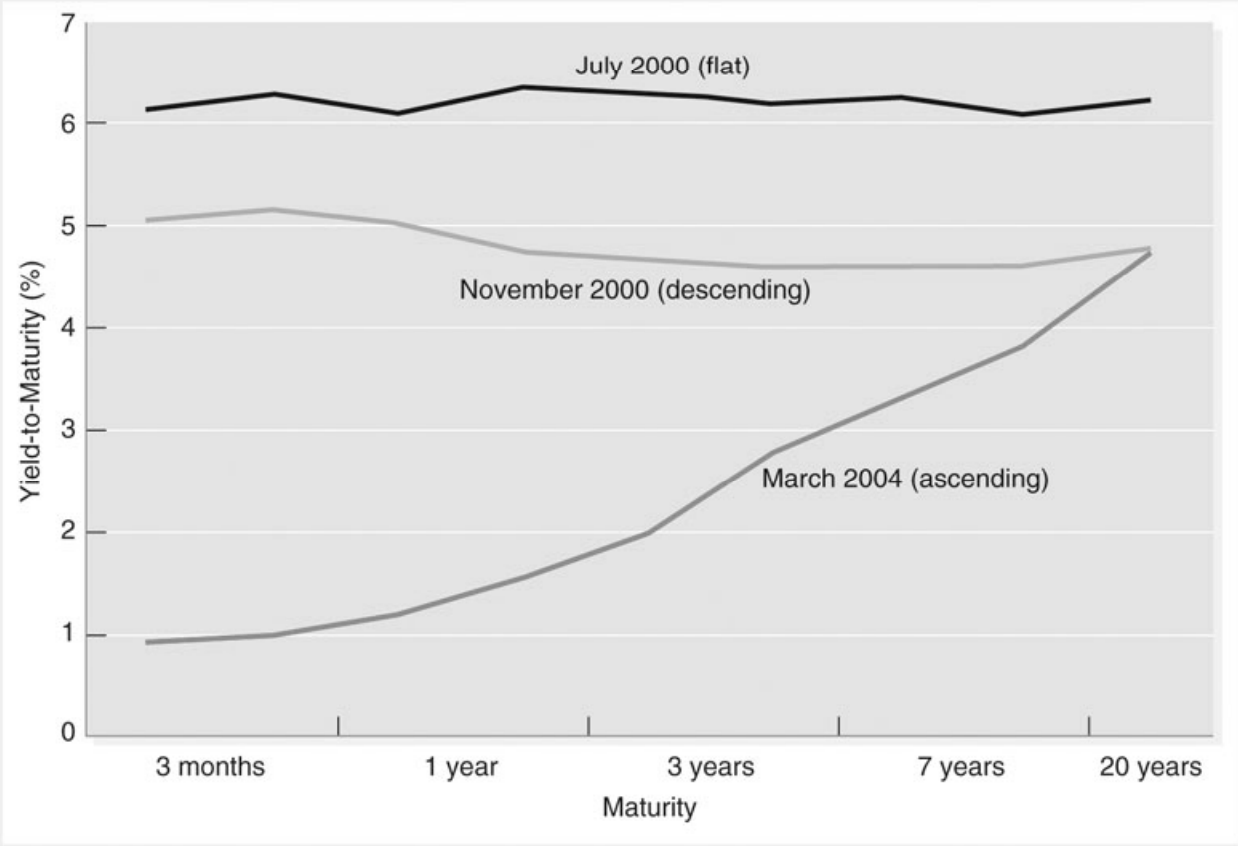
- Each theory has some element of truth
  - Day-to-day changes in the term structure suggest evidence of Preferred Habitat Theory.
  - Over many days, expectations of future interest rates and liquidity premiums are important components of the position and shape of the yield curve.

- Yields curves and business cycle
  - Ascending yield curve tends to indicate market expectations of economic expansion and/or inflation.
  - Descending yield curve tends to indicate lower short-term rates possibly related to slower economic growth or lower inflation rates.

- Security markets respond to updated new information and expectations and reflect their reactions in security prices and yields.

Yield Curves in the 2000s - Exhibit 6.1

EXHIBIT 6.1
Yield Curves on Treasury Securities in the 2000s

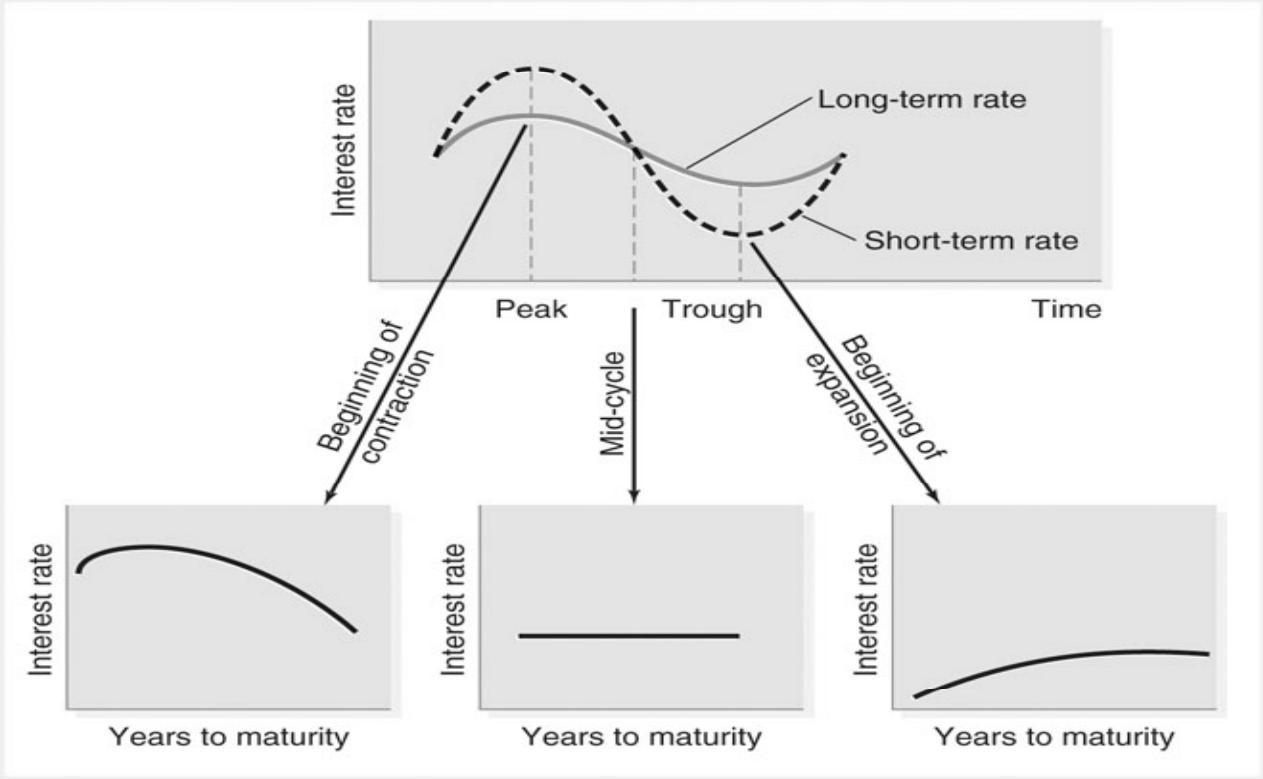


Although yield curves can be upward or downward sloping, they are typically upward sloping. There are also noticeable humps in the yield curves.

Source: Federal Reserve Board of Governors, H.15 Statistical Release.

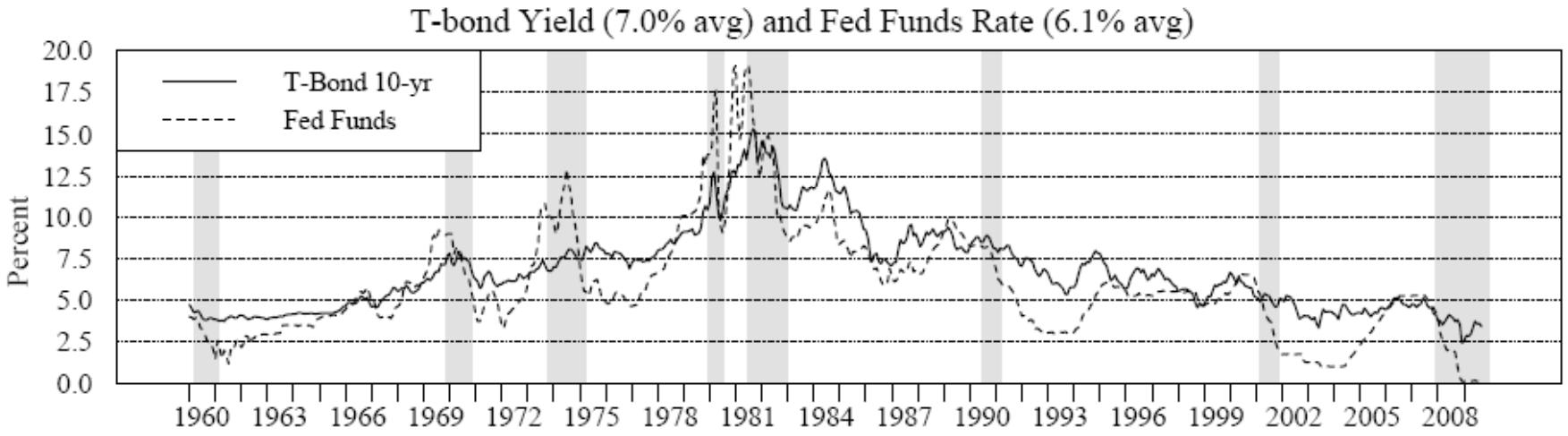
Yield-Curve Patterns Over the Business Cycle

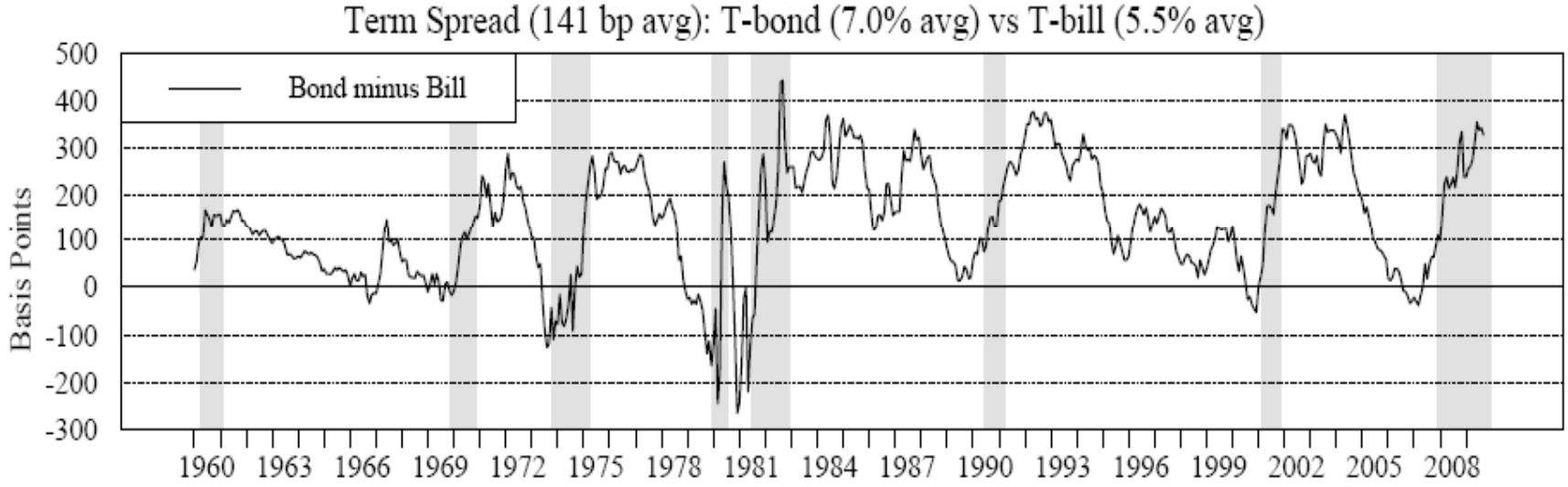
EXHIBIT 6.4
Interest Rate and Yield Curve Patterns over the Business Cycle



Yield curves are typically upward sloping during periods of economic expansion and turn downward when the economy begins to contract.

Historical Interest Rates / Slope of yield curve





Default Risk

- Default risk - the probability of the borrower not honoring the security contract
  - May range from “interest a few days late” to complete loss of principal.
  -

- Investors require default risk premium (above less risky securities) for added risk assumed. DRP = $i - i_{rf}$
  - Default risk premium (DRP) is the difference between the nominal rate and the yield on comparable (same term) riskless (Treasury) bond.

- Default risk premiums increase during recessions and decrease in expansions.
  - In good times, risky security prices are bid up, default premiums “compress.”
  - In bad economic environments,

- Credit rating agencies measure and grade relative default risk security issuers
  - Examine - Cash flow, level of debt, profitability, and variability of earnings.
  - Moody’s; Standard and Poors, Fitch.

Corporate Bond-Rating Systems, Exhibit 6.7

EXHIBIT 6.7
Corporate Bond Rating Systems

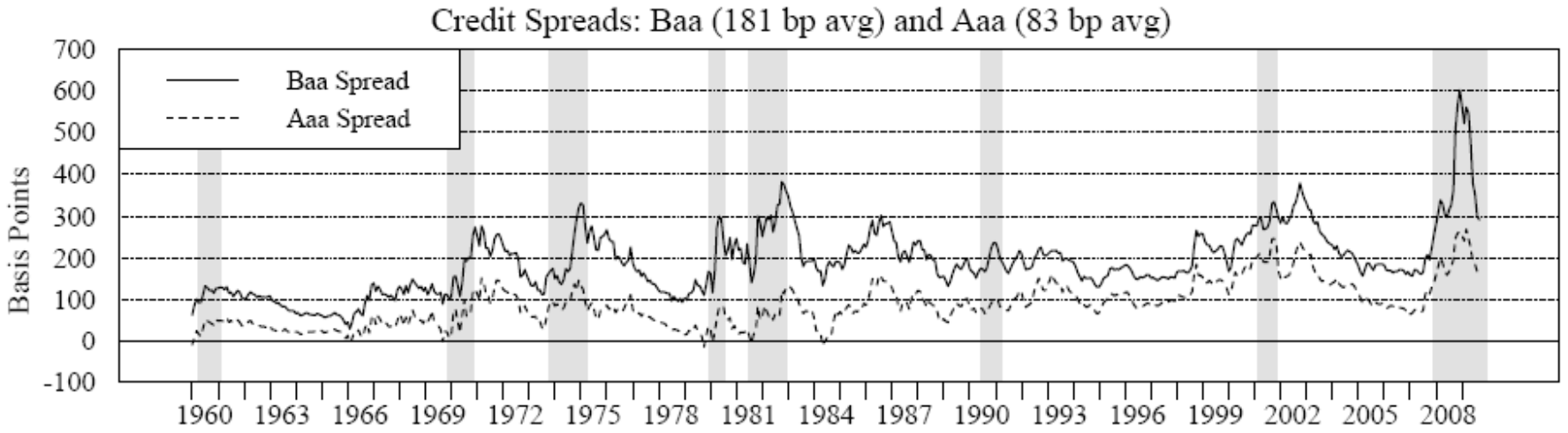
Explanation	Moody's	Standard & Poor's/Fitch	Default Risk Premium
Best quality, smallest degree of risk	Aaa	AAA	Lowest
High quality, slightly more long-term risk than top rating	Aa	AA	—
Upper-medium grade, possible impairment in the future	A	A	—
Medium grade, lack outstanding investment characteristics	Baa	BBB	—
Speculative issues, protection may be very moderate	Ba	BB	—
Very speculative, may have small assurance of interest and principal payments	B	B	—
Issues in poor standing, may be in default	Caa	CCC	—
Speculative in a high degree, with marked shortcomings	Ca	CC	—
Lowest quality, poor prospects of attaining real investment standing	C	C	Highest

Investment-grade bonds are those rated Baa or above by Moody's (or BBB by Standard & Poor's and Fitch). Bonds below Baa are speculative grade. Financial institutions are typically allowed to purchase only investment grade.

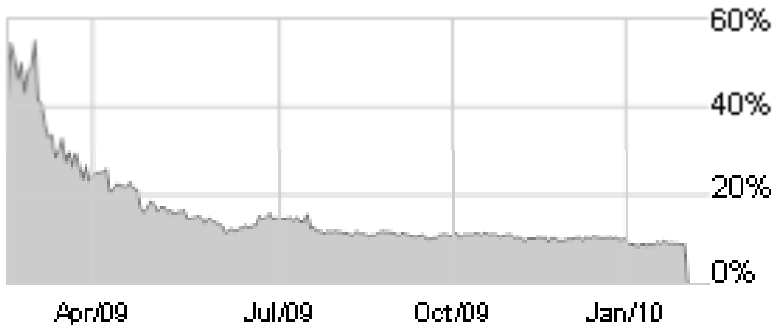
Note: Moody's applies the modifiers 1, 2, and 3 to the ratings Aa to Caa, with a 1 indicating the issue is in the higher end of its rating, a 2 indicating it is in the mid-range, and a 3 indicating it is in the lower end. Similarly, Standard & Poor's and Fitch modify their ratings AA to CCC with a + or – to indicate when an issue is in the higher or lower end of its rating category.

Default Risk Premiums (credit spreads)

- Default risk premium –
  - YTM on 10-yr Treas bond minus YTM on 10-yr Corporate (AAA / BAA)
  - Multiply by "100" to convert to "basis points"



- Ford 2025 bond with 7.125% coupon – yield and price



Risk Premiums (December 2006)

EXHIBIT 6.5

Risk Premiums for Selected Securities (December 2006)

Corporate Bonds	Security Yield (%)	Equivalent Risk-Free Rate (%)*	Risk Premium (%)
Aaa	5.70	4.56	1.14
Aa	5.90	4.56	1.34
A	6.10	4.56	1.54
Baa	6.50	4.56	1.94

*10-year Treasury note yield.

Source: Federal Reserve Statistical Release H.15 and Securities Industry and Financial Markets Association http://www.bondmarkets.com/story.asp?id=86.

Tax Effects and marketability

- Taxation – differential tax treatment on security gains and income affects the yield differences among securities
  - after-tax return, i_{at}: multiply pre-tax return by one minus investor's marginal tax rate: $i_{at} = i_{bt}(1-t)$
  - Municipal bonds' interest income is exempt from federal tax.
    - (and usually exempt from their own state tax)
  -

Investors' Marginal Tax Rate	Municipal Yield	Corporate After-Tax Yield
0%	7%	10 (1 – 0.00) = 10.0%
10	7	10 (1 – 0.10) = 9.0
20	7	10 (1 – 0.20) = 8.0
30	7	10 (1 – 0.30) = 7.0
40	7	10 (1 – 0.40) = 6.0
50	7	10 (1 – 0.50) = 5.0

Impact of Marketability on Interest Yields

- Marketability – The costs and speed with which investors can resell a security. Securities with good marketability have higher prices and lower yields.
  - Cost of trade.
  - Physical transfer cost.
  - Search costs.
  - Information costs.

- Auction rate securities are a type of bond which became extremely illiquid during 2008 crisis.
  -

Contract Options and Yields

- Various option provisions may explain yield differences between bonds.
  - An option is a contract provision which gives the holder or issuer the right, but not the obligation, to buy, sell, redeem, or convert an asset at some specified price within some specified time.

- A call option permits the issuer (borrower) to call (redeem) the bond before maturity at a prespecified price.
  - Borrowers call bonds if interest rates decline.
  - Investors in callable securities bear risk of losing their high-yielding bond.
  -
  - Your text calls this the *call interest premium* CIP = $i_c - i_{nc}$

- A put option permits the investor (lender) to sell the bond back to the issuer at a prespecified price before maturity
  - Investors are likely to put their bonds as interest rates increase.
  -
  - *put interest discount* PID = $i_p - i_{np}$

Contract Options and Yields

- Conversion option – permits the investor to convert a bond into another security (usually common stock)
  - Convertible bonds generally have higher prices (< YTMs) nonconvertibles.
- The *conversion yield discount* CYD = $i_{con} - i_{ncon}$.
-