# CHAPTER 5
# (part 2)

# BOND PRICES AND INTEREST RATE RISK

# *Chapter 5 (part 2) Outline*

- Yield measures
  - Yield to maturity
  - Expected yield
  - Realized yield
  - Total return

- Bond price volatility

- Interest rate risk – includes both reinvestment risk and bond price volatility
  - Duration
  - Properties of duration
  - Using duration to manage interest rate risk

# ***Common yield measures***

- Yield to Maturity – measures investor's expected return only if bond is held to maturity **and** all payments are reinvested at same yield!
  Equate original investment with the PV of payments (coupons and par value)
  –

- Expected Yield – Predicted yield for a given holding period less than maturity. Equate original investment with the PV of payments expected to be received.
  –

- Realized Yield - Investor’s *ex post* yield, given cash flows actually received and their timing. Differs from expected yield due to
  –
  –

- Total Return – investors total return (like modified IRR) found by computing FV of payments, after reinvestment.

# *Computing yields 1/2*

**YTM:** Investor buys 5% percent coupon (semiannual payments) bond for $951.90; bond matures in 3 years. Find the YTM.
Equate original investment with the PV of payments (coupons and par value)
A: 3.4% semiannually, or 6.8% annually. Excel: =RATE(6,25,-951.9,1000)

**Expected yield:** Suppose that the investor expects to sell the bond one year prior to maturity, for $975 (based on forecasted expected market conditions.)
Equate original investment with the PV of payments expected to be received.
A: 3.2% semiannually or 6.4% annually. Excel: =RATE(4,25,-951.9,975)

# *Computing yields 2/2*

- **Realized yield:** Suppose that one year prior to maturity, the investor *actually* sells the bond for $1020. Find the realized yield.
Equate original investment with the PV of payments received.
A: 4.3% semiannually or 8.6% annually. Excel: =RATE(4,25,-951.90,1020)

**Total Return:** Suppose that just after the first coupon is paid, interest rates move up to 10%. All coupons are reinvested at this 10% rate, and the bond is held to maturity. Find the total return.
Find FV of all cash flows. Equate original investment with this FV, discounted as a lump-sum.

A: 3.5% semiannually or 7.0% annually
Excel: =RATE(6,0,-951.90,1170.05)

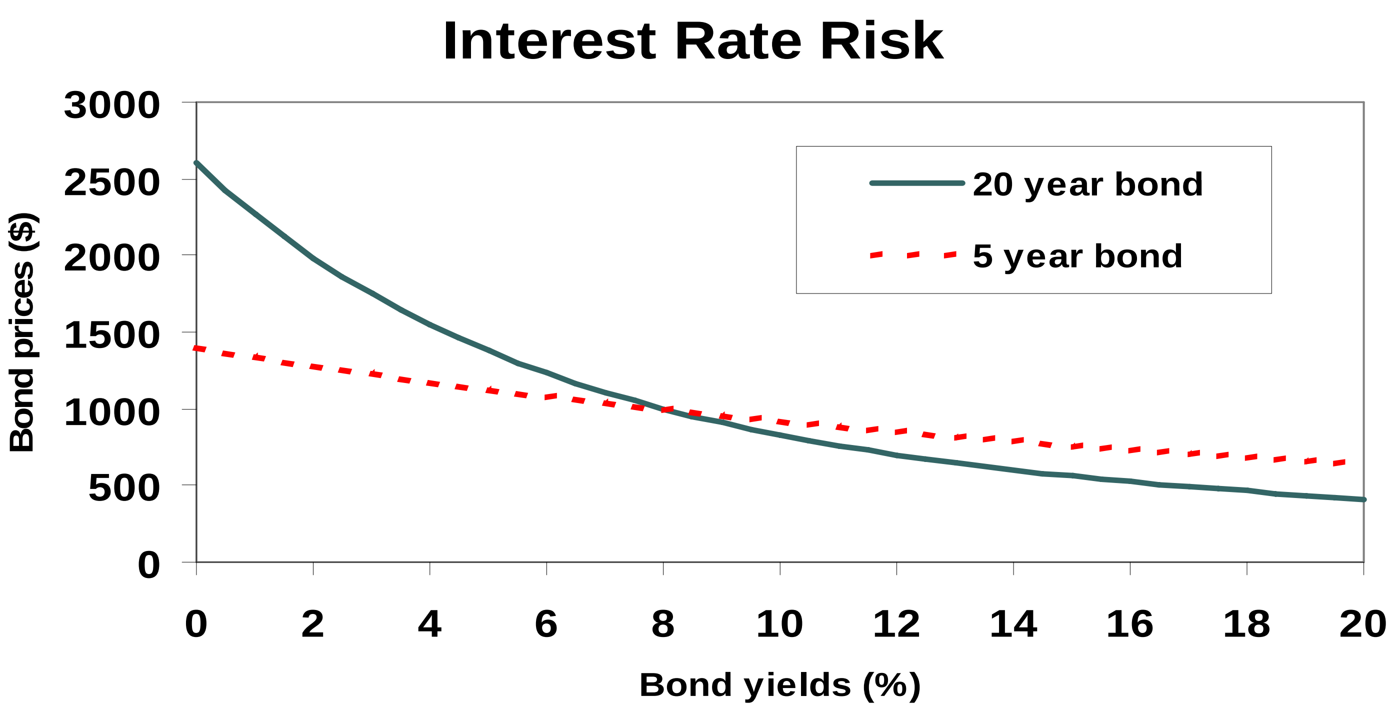
| PMT | FV |
|---|---|
| 25 | 31.91 |
| 25 | 30.39 |
| 25 | 28.94 |
| 25 | 27.56 |
| 25 | 26.25 |
| 1025 | 1025.00 |
| | **1170.05** |

# *Bond price volatility (price risk)*

Bond price volatility = Percentage change in price for given change in interest rates:

$$\%\Delta P_B = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100$$

where %ΔPB = percentage change in price
$P_t$ = new price in period t
$P_{t-1}$ = bond's price one period earlier



# *Interest Rate Risk and Duration*



- Price risk - is the variability in bond prices caused by their inverse relationship with interest rates.

- Reinvestment risk - is the variability in realized yield caused by changing market rates at which coupons can be reinvested.

- Price risk and reinvestment risk work against each other.
  - As interest rates fall —
    -
    -
  - As interest rates rise—
    -
    - .

- Interest rate risk – comprises both price risk and reinvestment risk.

# Bond price risk



EXHIBIT 5.1

**Relationship among Price, Maturity, Market Yield, and Price Volatility for a $1,000, 5% Coupon Bond (Annual Payments)**

| | | Price Change if Yield Changes to 6% | | | Price Change if Yield Changes to 4% | | |
|---|---|---|---|---|---|---|---|
| (1) Maturity (years) | (2) Bond Price at 5% Yield ($) | (3) Bond Price ($) | (4) Loss from Increase in Yield ($) | (5) Price Volatility (%) | (6) Bond Price ($) | (7) Gain from Decrease in Yield ($) | (8) Price Volatility (%) |
| 1 | $1,000 | $990.57 | $9.43 | –0.94 | $1,009.62 | $9.62 | 0.96% |
| 5 | 1,000 | 957.88 | 42.12 | –4.21 | 1,044.52 | 44.52 | 4.45 |
| 10 | 1,000 | 926.40 | 73.60 | –7.36 | 1,081.11 | 81.11 | 8.11 |
| 20 | 1,000 | 885.30 | 114.70 | –11.47 | 1,135.90 | 135.90 | 13.59 |
| 40 | 1,000 | 849.54 | 150.46 | –15.05 | 1,197.93 | 197.93 | 19.79 |
| 100 | 1,000 | 833.82 | 166.18 | –16.62 | 1,245.05 | 245.05 | 24.50 |

This exhibit shows that the longer the maturity of a bond, the greater the bond's price volatility. Thus, long-term bonds have greater interest rate risk than short-term bonds.

EXHIBIT 5.2

**Relationship among Price, Coupon Rate, Market Yield, and Price Volatility for a $1,000, 10-Year Bond (Annual Payments)**

| | | Price Change if Yield Changes to 6% | | | Price Change if Yield Changes to 4% | | |
|---|---|---|---|---|---|---|---|
| (1) Coupon Rate % | (2) Bond Price at 5% Yield ($) | (3) Bond Price ($) | (4) Loss from Increase in Yield ($) | (5) Price Volatility (%) | (6) Bond Price ($) | (7) Gain from Decrease in Yield ($) | (8) Price Volatility (%) |
| 0% | $613.91 | $558.39 | $55.52 | –9.04% | $675.56 | $61.65 | 10.04% |
| 5 | 1,000.00 | 926.40 | 73.60 | –7.36 | 1,081.11 | 81.11 | 8.11 |
| 10 | 1,386.09 | 1,294.40 | 91.69 | –6.62 | 1,486.65 | –100.56 | 7.25 |

This exhibit shows that the lower the coupon rate of a bond, the greater the bond's price volatility. Thus, low-coupon bonds have greater interest rate risk than high-coupon bonds.

# Duration - *a measure of interest rate risk*

- **Macaulay's Duration –** can be used to measure interest rate risk.
  - The formula is ugly, but the concept easy.
  - Calculated as the weighted average of time until each payment is received
  - Weights are proportional to the PV of each CF
    - CFt = interest or principal payment at time t
    - t = time period in which payment is made
    - n = number of periods to maturity; i = the yield to maturity

$$D=\frac{\sum_{t=1}^{n}\frac{CF_t * t}{(1+i)^t}}{\sum_{t=1}^{n}\frac{CF_t}{(1+i)^t}} \qquad D=\sum_{t=1}^{n}\left(t*\frac{PV(CF_t)}{PB}\right)$$

- Q: Find duration of an investment that pays 500 in year 1 and 500 in year 2. The appropriate discount rate is 10%
  - *Find PV of CF*
  - Divide by price (PV) of bond
  - Multiply each by "t"
  - Sum up

| | | | **w(t)** | |
|---|---|---|---|---|
| **Yrs (t)** | **CF** | **PV** | **PV/Pr** | **t x w(t)** |
| 1 | 500 | 454.55 | 0.5238 | 0.52 |
| 2 | 500 | 413.22 | 0.4762 | 0.95 |
| | | 867.77 | | **1.48** |

# *Duration Example*

- Duration (Maculay) $$D = \sum_{t=1}^{n} \left( t * \frac{PV(CF_t)}{PB} \right)$$

- Q: Find duration for a 30 year zero (without Excel "duration" function).

$$D = T \times \frac{PV\ (CF_T)}{Price} = 30 \times 1 = 30$$

- Q: Find duration of 3 yr, 8% coupon bond selling at par (YTM=8% BEY)
  - •Find PV of CF (discount by $1.08^t$)
  - •Divide by price (PV) of bond
  - •Multiply each by "t"
  - •Sum up

| | | | w(t) | |
|---|---|---|---|---|
| **Yrs (t)** | **CF** | **PV** | **PV/Pr** | **t x w(t)** |
| 1 | 80 | | | |
| 2 | 80 | | | |
| 3 | 1080 | | | |
| | | | | |
| | | **Bond price** | | **Bond du** |
| | | | | |
| | | | | |

# *Duration: Example (3)*

- Q: Find the duration of a 3 yr, 11% coupon bond selling at par (YTM=11%).

| | | | w(t) | |
|---|---|---|---|---|
| **Yrs (t)** | **CF** | **PV** | **PV/Pr** | **txw(t)** |
| 1 | 110 | 99.10 | 0.0991 | 0.10 |
| 2 | 110 | 89.28 | 0.0893 | 0.18 |
| 3 | 1110 | 811.62 | 0.8116 | 2.43 |
| | | 1,000.00 | | **2.71** |
| | | **Bond price** | | **Bond du** |
| | | | | |
| | | | | |

- Excel has nice `duration` and `mduration` functions!

# *Duration: Properties*

- Rule 1 The duration of a zero-coupon bond equals its time to maturity
- Rule 2 A bond's duration increases as its coupon rate decreases
- Rule 3 A bond's duration increases as its maturity increases
- Rule 4 A bond's duration decreases as its yield increases

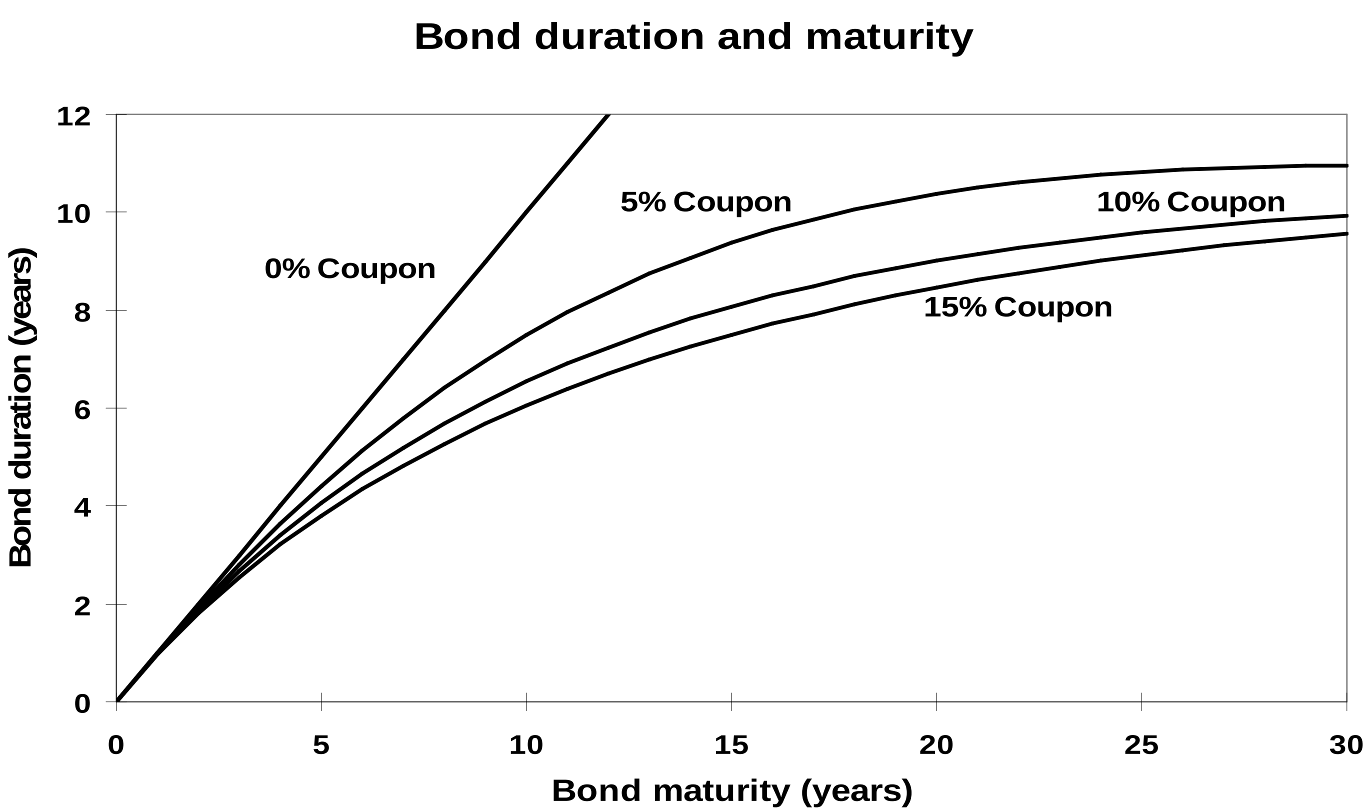


EXHIBIT 5.4

**Duration for Bonds Yielding 10% (Annual Compounding)**

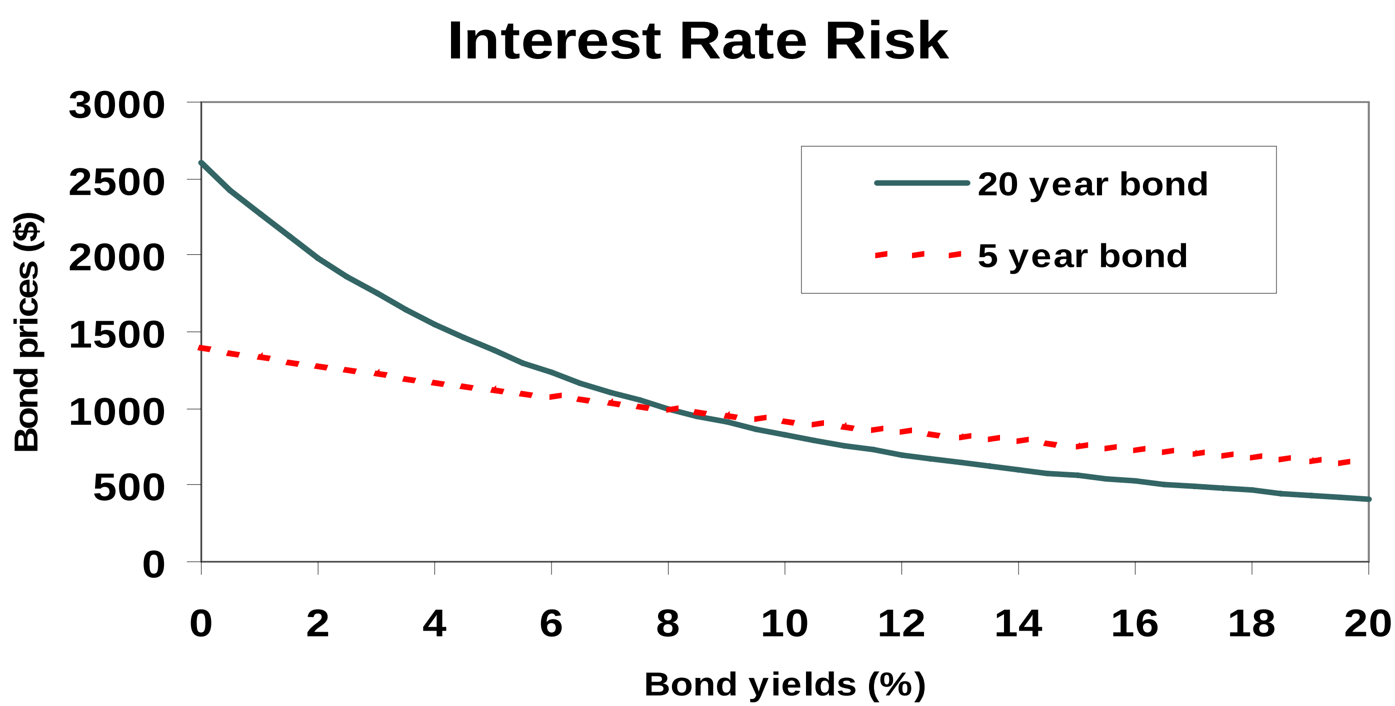
| | Duration in Years | | |
|---|---|---|---|
| Maturity (Years) | Zero Coupon | 4% Coupon | 8% Coupon |
| 1 | 1.00 | 1.00 | 1.00 |
| 2 | 2.00 | 1.96 | 1.92 |
| 3 | 3.00 | 2.88 | 2.78 |
| 4 | 4.00 | 3.75 | 3.56 |
| 5 | 5.00 | 4.57 | 4.28 |

*Duration* is a measure of bond price volatility that considers both the coupon rate and term to maturity.

# *Duration and Interest Rate Risk*

- Recall that duration helps measure "interest rate risk"
  - Percent ΔP with respect to ΔYTM is proportional to duration!
  - That is, we are interested in (ΔP/P) / Δi
  - This is about equal to the "derivative" (1/P)*dP/di

- **%ΔP/P = – D x Δi/(1+i)x100**



# *Interest Rate Risk: Example (1)*

- Q: Consider a 10 yr zero coupon bond with a 5% YTM. What is the effect on price of a 1% increase in the YTM?

  Recall **$\%\Delta P/P = -D \times \Delta i/(1+i) \times 100$**

  $\Delta P/P =$

- Note: Recall that for a zero, duration equals maturity. Therefore, for a zero, the $\%\Delta P/P$ due to a 1% $\Delta i$ is *about* equal to its maturity.

# *Interest Rate Risk: Example (2)*

• Q: Consider a bond selling at par with duration of 8.5 years and YTM of 9%. What is the effect on the bond price if the YTM goes to 11%.

Note: This bond (9% coupon, selling at par, duration 8.5) has ~15 year maturity.

•Recall %**ΔP/P = – D x Δi/(1+i)x100**

ΔP/P =

•Approx new bond price

•Note: Actual change in the bond's price (by computer) is –17.26%

•Note: The duration of a portfolio is the weighted avg of the duration of the individual bonds!

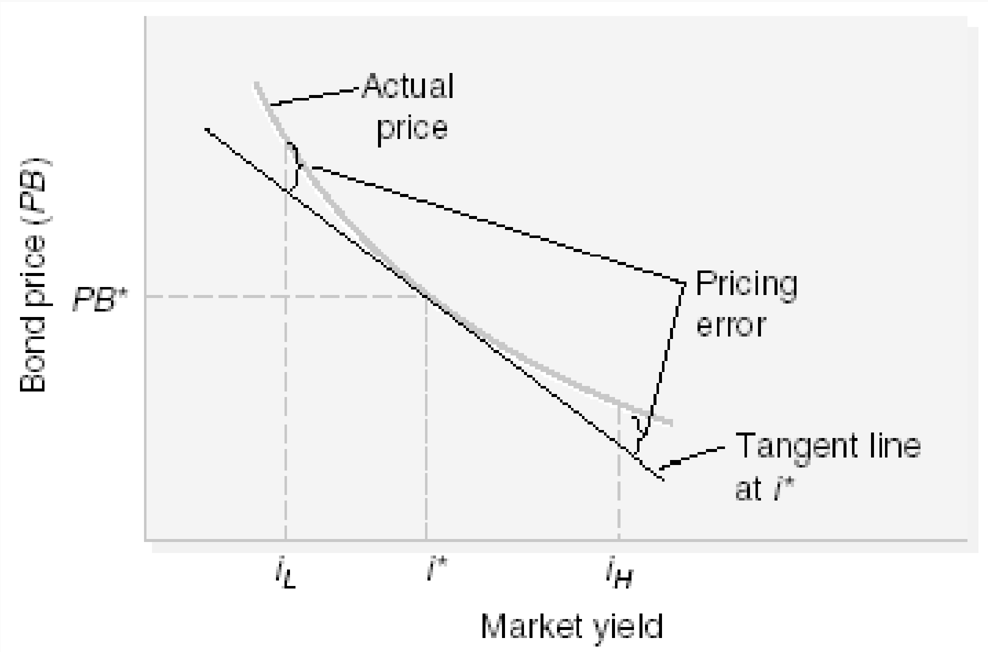
$$\textbf{\textit{Portfolio Duration}} = \sum_{n}^{i=1} w_i D_i$$

# *Duration is an approximation*

- Duration gives the "approximate" change in price for a given change in yield
  - It approximates a non-linear function with a linear function.
  - In advanced courses, you learn about "convexity" correction.

EXHIBIT 5.5

**The Typical Price–Yield Relationship**




The tangent line to the price-yield profile can be used to estimate changes in bond prices due to changes in interest rates.

# *Using Duration to Manage Interest Rate Risk*

- Duration (alternative description): the holding period for which reinvestment risk just offsets price risk. The holder obtains the original, "promised" YTM.
- Uses of duration – Financial institutions use duration to manage interest rate risk and achieve the desired yield for the desired holding period.
  - Imagine a pension fund (or insurance co) that sells contract obligating them to make payments in future given premiums collected today.
  - 
- ***Maturity matching***: Selecting a term to maturity equal to the desired holding period eliminates price risk, but not reinvestment risk.
- ***Zero-coupon approach***: zero-coupon bonds have no reinvestment risk.
  - The duration of a "zero" equals its term to maturity.
  - Buy a "zero" with the desired holding period and lock in the YTM.
  - Must hold to maturity to evade price risk.
- ***Duration matching***: To realize yield to maturity, investors select bonds with durations matching their desired holding periods.

# *Duration Matching - example*



- Insurance company hedges a liability by buying a bond. The liability is a three year guaranteed investment contract.
  - Liability: GIC has PV=10,000; YTM=8%. FV=$10,000(1.08)$^5$=$14,693
  - Asset: Company invests in $10,000 par bond; T=10 yr; Annual coupon=8%.
    - Bond sold after five years
    - If rates remain at 8%, liability is offset.
    - If rates change immediately, will liability be funded?

- Interest rate volatility results in both *price risk* **and** *reinvestment risk !!*

| **Rates remain at** | | | **8%** | **7%** | **9%** |
|---|---|---|---|---|---|
| **Time** | **Yrs left** | **CF** | **Value at T=5** | | |
| 1 | 4 | 800 | 1,088.39 | 1,048.64 | 1,129.27 |
| 2 | 3 | 800 | 1,007.77 | 980.03 | 1,036.02 |
| 3 | 2 | 800 | 933.12 | 915.92 | 950.48 |
| 4 | 1 | 800 | 864.00 | 856.00 | 872.00 |
| 5 | 0 | 800 | 800.00 | 800.00 | 800.00 |
| 5 | 0 | Remaining | 10,000.00 | 10,410.02 | 9,611.03 |
| | | | | | |
| **Sum of payments** | | | **14,693.28** | **15,010.61** | **14,398.80** |
| **Duration** | | | **7.25** | **7.35** | **7.15** |

# *Duration Matching - example*



- Insurance companies – Consider the following
  - Liability: GIC has PV=10,000; YTM=8%. FV=$10,000(1.08)^5=$14,693
  - Asset: Company invests in $10,000 par bond; T=6 yr; Annual coupon=8%.
    - Bond sold after five years
    - If rates remain at 8%, liability is offset.
    - If rates change immediately, liability is still funded!

- Duration of assets matches liabilities, so price risk cancels reinvestment risk! (except for convexity)

| **Rates remain at** | | | **8%** | **7%** | **9%** |
|---|---|---|---|---|---|
| **Time** | **Yrs left** | **CF** | **Value at T=5** | | |
| 1 | 4 | 800 | 1,088.39 | 1,048.64 | 1,129.27 |
| 2 | 3 | 800 | 1,007.77 | 980.03 | 1,036.02 |
| 3 | 2 | 800 | 933.12 | 915.92 | 950.48 |
| 4 | 1 | 800 | 864.00 | 856.00 | 872.00 |
| 5 | 0 | 800 | 800.00 | 800.00 | 800.00 |
| 5 | 0 | Remaining | 10,000.00 | 10,093.46 | 9,908.26 |
| | | | | | |
| **Sum of payments** | | | **14,693.28** | **14,694.05** | **14,696.03** |
| **Duration** | | | **4.99** | **5.02** | **4.97** |