CHAPTER 5

BOND PRICES AND INTEREST RATE RISK

Future Value or Compound Value

- Future value (FV) - Let *i* be (constant) periodic interest rate; *n* is number of compounding periods. The FVof a sum (PV) is

$$\mathbf{FV = PV\ (1+i)^n}$$

- Q: What is FV of $1,000 to be received in 1 year. The interest rate is 10% / yr.

- Present Value - The value now of a sum expected at a future time

$$PV = FV \frac{1}{(1+i)^n}$$

- Q: What is PV of $1,100 to be received in 1 year? The interest rate is 10% / yr.
-

- Risk: The interest , or discount, rate should reflect the riskiness of cash flows.
  - If the cash flow is riskier, the discount rate should increase, and PV will fall.

Discounting Annuities

- Annuities -annuities are streams of fixed payments at fixed intervals for a fixed period. Annuities are common in finance.
  - Bonds, and some stocks, look at lot like an annuity.
  - Most loans (car loans, mortgages, student loans) are amortized as annuity;
  - Insurance policies and retirement plans are also types of annuities.

Find the PV of $1/yr to be received in the next five years at discount rate of 10%				
0	**1**	**2**	**….**	**5**
???	1	1	….	1

PV =

How about the PV of an annuity over 100 years?

$PV = C\,[\,1/(1+i) + 1/(1+i)^2 + \cdots + 1/(1+i)^n\,]$

$PV = 1/1.1 + 1/1.1^2 + 1/1.1^3 + \ldots + 1/1.1^{99} + 1/1.1^{100}$

Discounting Annuities

- To find the PV of a stream of cash flows, we can just discount each payment:

$$PV = \frac{C_1}{(1+i_1)} + \frac{C_2}{(1+i_1)(1+i_2)} + \dots + \frac{C_n}{(1+i_1)(1+i_2)\dots(1+i_n)}$$

- Simplification #1 – the interest rate is constant

$$PV = \frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \dots + \frac{C_n}{(1+i)^n}$$

- Simplification #2 – the payments are constant

$$PV = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \dots + \frac{C}{(1+i)^n}$$

- The miracles of modern algebra allows us to simplify line #2
  - The PV of an annuity payment “C” for “T” years and beginning in 1 year:

$$PV = C\frac{1-\left(\frac{1}{1+i}\right)^n}{i}$$

Discounting Annuities

- How about the PV of an $1 annuity over 100 years?

$$PV = \frac{1}{(1.1)^1} + \frac{1}{(1.1)^2} + \ldots + \frac{1}{(1.1)^{100}}$$

Discounting Annuities

Suppose that you win a $1M lottery, but instead of paying $1M, you receive $50,000/yr for 20 years ! What is the true value of this lottery? (Assume the appropriate discount rate is 8%.)

0	**1**	**2**	**…**	**20**
???	50,000	50,000	…	50,000

$$PV = 50{,}000\ (1/1.08 + 1/1.08^2 + 1/1.08^3 + 1/1.08^4 + \ldots + 1/1.08^{20})$$

PV

$PV_A =$

- Perpetuities: What is the PV of $1/yr forever? Or the PV of $50,000/yr forever?
  - Recall the PV of $1/yr for 100 years at r=10% was 9.9993.

- In general, the PV of $C/year forever is C/*i*.

- The PV of $1/year at 10% is
- The PV of $50,000/yr at 10% is

DCF: Financial Calculator

- Financial calculators have powerful financial functions.
  - Can be used to find the PV of annunities!

- Setting up the HP10B
  - Clear registers: CLEAR ALL
  - Setting compounding freq to 1x/period: 1 SHIFT P/YR
  - Set payments to end of period: BEG/END ("begin" not in display).

- Setting up the TI-BAII+
  - 2nd CLR TVM
  - Set compounding freq to 1x/period: [2nd] P/Y, then 1 ENTER ↓ 1 ENTER.
  - Set payments to end of period [2nd] BGN [2nd] SET ("begin" not in display).

Data	**Button**
50,000	PMT
20 (number of periods)	N
8 (periodic interest rate)	I/Y or I/YR
compute PV (TI BAII Plus)	**CPT PV**
compute PV(HP10B)	**PV**

Excel

- MS Excel Spreadsheet
  - Is very, very powerful!

- Use help menus to find financial functions with examples
  - Select "F1" for "Help"
  - Select "Function Reference"; then "Function by category"
  - Scroll to "financial functions"
  - Select "PV" to find present value.

PV(**rate**,**nper**,**pmt**,fv,type)		
PV(10%,100,1)		
(9.9993)		

Suppose we knew that the PV was 9.9993.				
Could we work backward to find the "discount" or "interest" rate?				
RATE(**nper**,**pmt**,**pv**,fv,type,guess)				
RATE(100,1,-9.9993)				
10.00%				

Bond Pricing: What is a bond?

- Bond – is a form of loan. It is a debt security obligating a borrower to pay a lender principal and interest.
  - Borrower (*issuer*) promises contractually to make periodic payments to lender (*investor* or *bondholder*) over given number of years.

- Lender (bond holder) receives
  - Annuity – periodic interest (*coupon*) payment determined by *coupon rate.*
  - Lump-sum (at maturity)) – .

  - The annuity is determined by the coupon rate, which is the original "interest rate" promised as percentage of par on face of bond.

- Price (value) of bond – is the present value of the promised future cash flows, discounted at the market rate of interest (the required rate of return on this risk class in today's market)
  - For a bond, this discount rate is often called the "Yield to maturity".
  -

Bond – simple example

PV(rate,nper,pmt,fv,type)		
PV(5%,30,50,1000)		
(1,000.00)		

Par value	$1,000
Coupon Rate	5%
Issued	Today
Matures	30 years from today
Payments:	$50/year interest for 30 years; $1,000 par at end of year 30

For simple bond problems, we can use Excel PV function or calculator.

- Q: if the discount rate (or YTM) is 5%, what is the price?
- A: Price =

- Q: if the discount rate (or YTM) is 10%, what is the price?
- A:

- Q: if the disc

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$$PV = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \ldots + \frac{C}{(1+i)^n} + \frac{F_n}{(1+i)^n}$$

$$PV = C\left(\frac{1-\left(\frac{1}{1+i}\right)^n}{i}\right) + \frac{F_n}{(1+i)^n}$$

More on Yield to Maturity

- YTM – rate that equates bond price with present value of all payments. Prices and YTMs are inversely related.
  - If YTM > coupon, then price < par and bonds sells at <u>discount</u>. Investor gets "capital gain" at maturity.
  - If YTM < coupon, then price > par and bonds sells at <u>discount</u>. Investor takes "capital loss" at maturity.
- Finding the YTM – this can't be solved for algebraically, but we can
  - (1) Guess; (2) use Excel; or (3) use financial calculator
- Q: If the price on the previous bond 900, what is the YTM?
  - Q2: What if the price was 1,100?
- A1: .
- A2:

$$PV = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \dots + \frac{C}{(1+i)^n} + \frac{F_n}{(1+i)^n}$$

RATE(**nper**,**pmt**,**pv**,fv,type,guess

RATE(30,50,-900,1000)		
5.70%		

Bond – Real world Example

Bond	**Cur Yld**	**Vol**	**Close**	**Net Chg**
F 7.125 25	10.18%	---	70.00	-0.94

Coupon = 7.125% of par annually (usually assume half is paid semi-annually)
Par = 1,000 (assume) 2025 = Maturity
Cur Yld = (annual coupon / price) Net Chg = change from previous day
Close = price at market close. http://cxa.marketwatch.com/finra/MarketData/

Time (1/10)	**0**	**0.5**	**1**	**1.5**	**…**	**15**
CF	**700.00**	35.625	35.625	35.625	…	35.625+1000

- The bond has an "annuity" plus a lump sum payment (par value).
  - The relationship between price and YTM (discount rate) is given below.

$$PV = \frac{C/m}{(1+i/m)^1} + \frac{C/m}{(1+i/m)^2} + \dots + \frac{C/m}{(1+i/m)^{mn}} + \frac{F_n}{(1+i/m)^{mn}}$$

Bond Pricing

- Q: What is the price of the bond at a discount rate (YTM) of 11.33%?

$PV =$

$PV =$

- Q: What if the discount rate (yield) is 7.125%?
- A:

- With this example, we can still use the "PV" function

=PV(11.33%/2,30,71.25/2,1000)		
(699.92)		

=PV(11.33%/2,30,71.25/2,1000)		
(534.97)		

Bond Pricing with Calculator

- Setting up the HP10B
  - Clear registers: CLEAR ALL
  - Setting compounding freq to 1x/period: 1 SHIFT P/YR
  - Set payments to end of period: SHIFT BEG/END ("begin" not in display).

- Setting up the TI-BAII+
  - 2^{nd} CLR TVM
  - Set compounding freq to 1x/period: [2^{nd}] P/Y, then 1 ENTER ↓ 1 ENTER.
  - Set payments to end of period [2^{nd}] BGN [2^{nd}] SET ("begin" not in display).

Data	**Button**
–700.00 (current price)	PV
32 (number of periods)	N
1000 (future value	FV
35.625	PMT
compute yield (TI BAII Plus)	CPT I/Y
compute yield (HP10B)	I/YR

Your answer will be the semi-annual yield (multiply by 2 to get BEY)

Bond Pricing with Excel

- More advanced bond pricing functions are "yield" and "price"
  - They account uneven number of days…
- Be sure to load Analysis Toolpak! (may have to restart Excel after loading.)
  - **Excel 03:** Start Excel, click "Tools" "Add-ins" check "Analysis Toolpak".)
  - **Excel 07:** search help for "Load the Analysis ToolPak"
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Bond Pricing		**Yields**	**Prices**
MDuration	**7.85**		
YTM (BEY)		**11.33%**	11.330%
Pr (Price)	70.00		**69.992**
Settlement	1/15/2010		
Maturity	1/15/2025		
Rate (Coupon rate)	7.125%		
Redemption (Maturity Value)	100		
Frequency (Coupons per yr)	2		
=YIELD(settlement,maturity,rate,pr,redemption,frequency,basis)			
=PRICE(settlement,maturity,rate,yld,redemption,frequency,basis)			

Factors Affecting Bond Yields 1/2

Anything that affects risk or timing of cash flows will affect bond yields

- Default risk – Corporate bonds have higher risk of default, so higher offer higher yields.

  Character –

  Investment Grade: S&P: AAA, AA, A, BBB (Moody: Aaa, Aa, A, Baa). BBB – normally adequate capacity to repay, but subject to adverse economic conditions or changing circumstances.

  Speculative, Junk – BB, B, CCC, C, D (M: Ba, B, Caa, C, D). BB – predominantly speculative capacity to repay.

Factors Affecting Bond Yields 2/2

- Reinvestment risk – what rate will coupons be reinvested?
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- Interest rate risk – risk that changing yields cause bond prices to fall.
  - increases with longer maturity and lower coupons (more later).
- Other factors affecting yields
- Taxes – Interest on treasuries not subject to state and local tax.
  -
- Maturity – bonds with short maturities usually have lower yields (more later).
  -
- Embedded options – many corporate bonds have a call provision.
  - Call price usually starts as par plus one coupon, then converges to par.
  - Initial period often *call protected.* Bonds usually trade less than call price.

Historical Yields