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Exotic Options and other Derivatives

- Exotic Options
  - Packages
  - Time dependent
  - Path dependent
  - Valuation

- Mortgage backed securities

- Non-standard swaps
  - Accreting swap, Amortizing swap, Basis swap, Compounding swaps, Currency swap, Roller coaster swap, Seasonal swap, Commodity swaps and others….

2 Exotic Options

- Packages – portfolio of standard Euro calls, puts, forwards cash and underlying. Often structured to be zero-cost. E.g, package may be constructed so that underlying can be sold for price between lower and upper bound.

- Compound options – European options on another options (e.g., call on call, call on put, etc.). Has two strike prices and expiration dates.

- Time-dependent options
  - Nonstandard American – early exercise restricted or strike price change.
  - Chooser options -
  - Forward start options – option that start at some time in the future, such as employee incentive schemes.
  - Bermuda option –

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Exotic Options – Path-Dependent 1

- Barrier options - payoffs based on “knock-in or knock-out” determined by underlying hitting either high or low extreme (trigger) during the contract. Frequency of price sampling specified in contract. Some have neg vega!
  - knock-in option - payout locked-in when underlying factor hits trigger. If trigger is not hit, option expires worthless.
  - knock-out option - option expires worthless when underlying factor hits trigger. Payoffs is same as standard option as long as trigger is not hit.
- Barrier option variations -
  - inside barrier - payout and triggers determined by price of same underlying.
  - outside barrier - trigger based on one factor and payout based on another.
  - double-barrier - one trigger above current price and one trigger below.
  - Roll-up put - like standard put until a trigger is reached. Then converted to an up-and-out put with the original trigger as the new strike price and a higher trigger level established above that

4 Exotic Options – Path-Dependent 2

- Lookback options - payoff based on highest or lowest price experienced during the lookback period, whichever most advantages to option holder. Frequency of sampling underlying price is specified in contract. Often on commodities.
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- Ladder options - strikes that are updated and reset if certain triggers are hit.
  – Call option X=$30, with ladder calling for strike reset at each 10 point trigger.
  – If underlying price hits $40, new strike becomes $40 and $10 gain locked-in.
  – If price equals $45 at expiration, holder receives $10 locked-in, plus $5.
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- Shout options – similar to ladder options, except holder may determine ladder or higher strike price by "shouting" when underlying reaches level he wishes to lock in.

- Asian options - based on average value of underlying factor over a specified portion of the contracts life.

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Exotic Options – Valuation 1

- Valuing Exotic Options with path dependence
  - Black-Scholes is not appropriate
  - Binomial will not work – e.g., for Asian option, how do we interpret average price? How do we interpret barrier?
  - We need a method that simulates actual price paths.

- Valuing Exotic Options by Monte Carlo Simulation – 4 steps
  - (1) Specify stochastic process (or distribution) describing possible realizations for underlying factor.
  - (2) Use computer program to generate possible realizations for underlying.
  - (3) Compute value of derivative based on realizations of the underlying.
  - (4) Return to step 2 many times.

- Example of Monte Carlo - Suppose European put option has strike price of $100 and matures in 5 days. The current price of underlying share is $100, with std dev of is 35%. Risk-free rate is 5%. The stock pays no dividends.
  - (1) Assume stock price distributed log-normal.

$$S_T = S_0 e^{\left(r - \frac{s^2}{2}\right)T + s\sqrt{T}Z}$$

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Exotic Options – Valuation 2

- Example, cont'd
  - (2) Use computer program to generate possible realizations for underlying. Five "draws" from std normal: {–0.7341, 0.2143, 0.7968, 0.4544, –0.9235}. Sequence of share prices is then:

$$S_1 = 100e^{\left(0.05-\frac{0.35^2}{2}\right)\frac{1}{252}+0.35\sqrt{\frac{1}{252}}(-0.7341)} = 98.3902$$

$$S_2 = 98.3902e^{\left(0.05-\frac{0.35^2}{2}\right)\frac{1}{252}+0.35\sqrt{\frac{1}{252}}(0.2143)} = 98.8518$$

$$S_3 = 98.8518e^{\left(0.05-\frac{0.35^2}{2}\right)\frac{1}{252}+0.35\sqrt{\frac{1}{252}}(0.7968)} = 100.5993$$

$$S_4 = 100.5993e^{\left(0.05-\frac{1}{2}0.35^2\right)\frac{1}{252}+0.35\sqrt{\frac{1}{252}}(0.4544)} = 101.6077$$

$$S_5 = 101.6077e^{\left(0.05-\frac{1}{2}0.35^2\right)\frac{1}{252}+0.35\sqrt{\frac{1}{252}}(-0.9235)} = 99.5552$$

- Example, cont'd (note: riskless rate can be used under risk-neutral pricing).
  - (3) Compute value of derivative based on realizations of the underlying. Value at expiration is P_T =
    Value today is $P_0 = F_T/(1+r)^T$ =

7 Exotic Options – Valuation 3

- Example, cont'd
  - (4) Return to step 2 many times. After 1,000 iterations, average p = 1.9928. This is fairly close to Black-Scholes price, which is \$1.9165! Why?

t	Z_t	S_t	F_T	$F_0=F_T/(1+r)^T$
0	0	100.0000		0.4444
1	-0.7341	98.3902		
2	0.2143	98.8518		
3	0.7968	100.5993		
4	0.4544	101.6077		
5	-0.9235	99.5552	0.4448	

Iterations	**Value**
1	0.4444
10	0.8510
100	1.7548
1,000	1.9928
10,000	1.9459
100,000	1.9068
Black-Scholes	1.9165

- Example, Asian option : Suppose this put option were an Asian put option, where payoff depends on arithmetic average of underlying share price.
  - (3) Compute value of derivative based on realizations of underlying.
    Avg S_T = (100.00 + 98.39 + 98.85 + 100.59 + 101.60 + 99.55)/6 = \$99.834
    $F_0 = F_T/(1+r)^T$ = (\$100 – \$99.8340) / $1.05^{5/252}$ = \$0.1659
  - (4) Return to step 2 many times. 1,000 iterations gives avg p_{Asian} = \$1.0578.

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Mortgage Backed Securities

- Mortgage holder has call option on loan – by convention US mortgage holders can pre-pay mortgages early. The is similar to a callable corporate bond. Lenders need to value this prepayment option. Accelerated securitization in 1960's offered many advantages:
  - Regional lending institutions and bond investors diversify.
  - Investors access more liquid bonds, due to standardization.
  - Home buyers have access to national capital market.

- Pass-through (mortgage) certificates – bonds issued, often by GNMA, FMNA, FRE, securitized by underlying mortgages.

- Collateralized Mortgage Obligations - Securities (bonds) created by splitting a mortgage pool cash flows according to specific allocation rules. Usually creates some securities with more/less prepayment risk.
  - Sequential CMOs – All payments to principal go to A tranche…
  - Protected amortization class (PACs) – PAC not affected by collar prepays.
  - Interest-only strip (IO) –
  - Principal-only strip (PO) –

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Nonstandard Swaps 1

- Accreting swap (step-up) –notional principal value increases over the life of the swap, which increases the relative size of the cash flows over time.
- Amortizing swap –notional principal value declines over life of swap, which decreases the relative size of the cash flows over time.
- Basis swap – Counterparties each make payment based upon one of two floating rates, e.g., LIBOR and T-bill. Aka floating-for-floating swap.
- Compounding swaps – swap payments compounded to maturity and netted.
- Currency swap – swap payments on s-t rates in different countries. Principal exchanged. Any combination of fixed-for-floating.
- Roller coaster swap –notional principal increases and then declines to zero.
- Seasonal swap – notional-principal value varies at based on seasonal schedule.
- Commodity swaps – swap $1M/yr for 100,000*spot price of commodity.

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Nonstandard Swaps 2

- Constant-maturity swap – special case of yield-curve swap where constant-maturity Treasury yield is paid by counterparties. E.g., one party pays constant maturity 5-year rate, other party pays constant maturity 90-day rate.
- Diff swap (rate differential swap) – all payments are made in common currency, but cash flows determined using prevailing interest rate in different countries.
- Index amortizing swaps – notional principal reduced by interest rate index.
- Yield-curve swap – variation on basis swap where one party pays floating rate based on T-bill rate and counterparty pays on movements in 30-yr T-bond rate.
  - Constant-maturity swap – each pays constant-maturity Treasury yield.
- Volatility swap – one party pays based on recently calculated daily return volatility in return for payment based on constant volatility.
- Equity swap – swap total return on equity index for either floating/fixed rate.

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Swaps combinations

- Forward swap – Cash flows begin at some specified future date.

- Swaption – option to enter a swap.
  - Payer – option to enter as fixed rate payer.
  - Receive – option to enter as fixed rate receiver.

- Cancelable – one party can cancel between effective and maturity date.

- Extendable – one party can extend swap beyond maturity date.

- Conclusion – There are many OTC variations on standard derivative contracts!