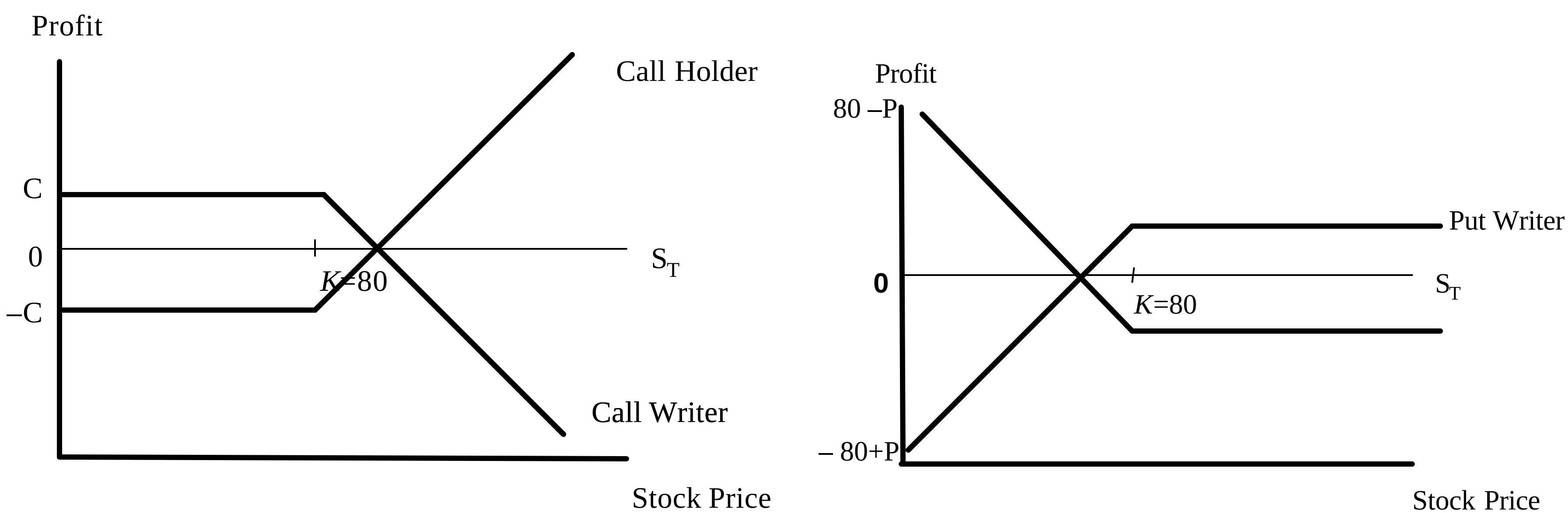
Options Properties

- Factors affecting Option Prices
  - Strike price, time, volatility, risk-free rate, dividend yield
- Assumptions and Notation
- Upper and Lower Bounds for Options Prices
- Put-Call Parity
  - Put-Call parity with no dividends $c_0 - p_0 = S_0 - Ke^{-rfT}$
  - Put-Call parity with known dollar div $c_0 - p_0 = S_0 - Ke^{-rT} - \text{PV}(Div)$
  - Put-Call parity with constant div yield $c_0 - p_0 = S_0 e^{-qT} - Ke^{-rT}$
- Early Exercise
  - calls on div paying stock
  - puts on a non-div paying stock
- Effects of Dividends
- Empirical Research

Notation

- Notation and Assumptions
  - c : European call option price C : American Call option price
  - p : European put option price P : American Put option price
  - S_0 : Stock price today K : Strike price
  - T : Life of option σ: Volatility of stock price
  - S_T : Stock price at option maturity D : PV of div during option's life
  - r : Risk-free rate for maturity T with cont comp
  - American option is worth at least as much as European $C \geq c; \quad P \geq p$
  - Assumptions for pricing options - No transactions costs; all profits taxed at same rate; borrow and lend at r

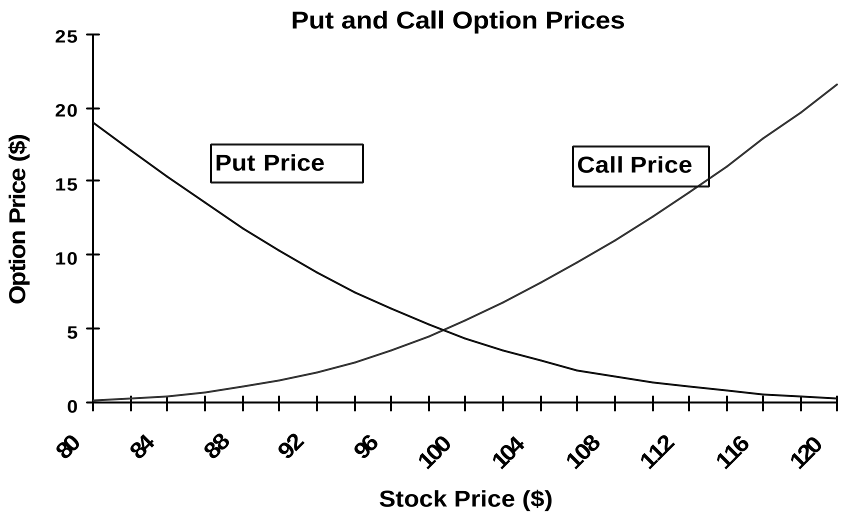


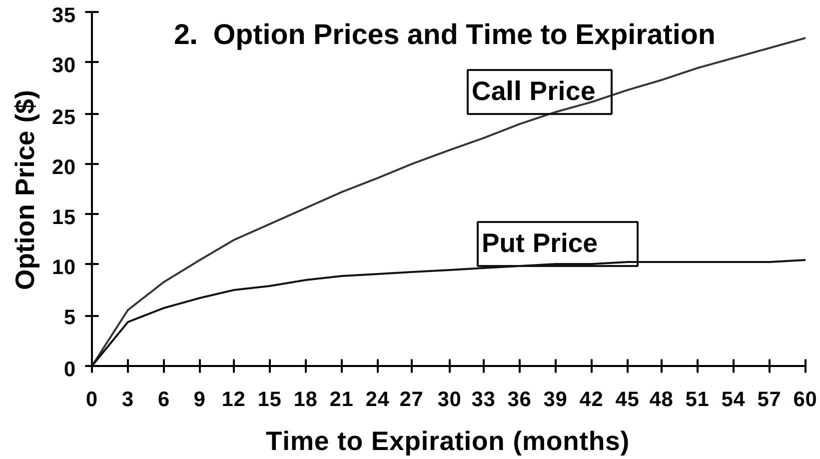
Sensitivity of Option Prices (a)

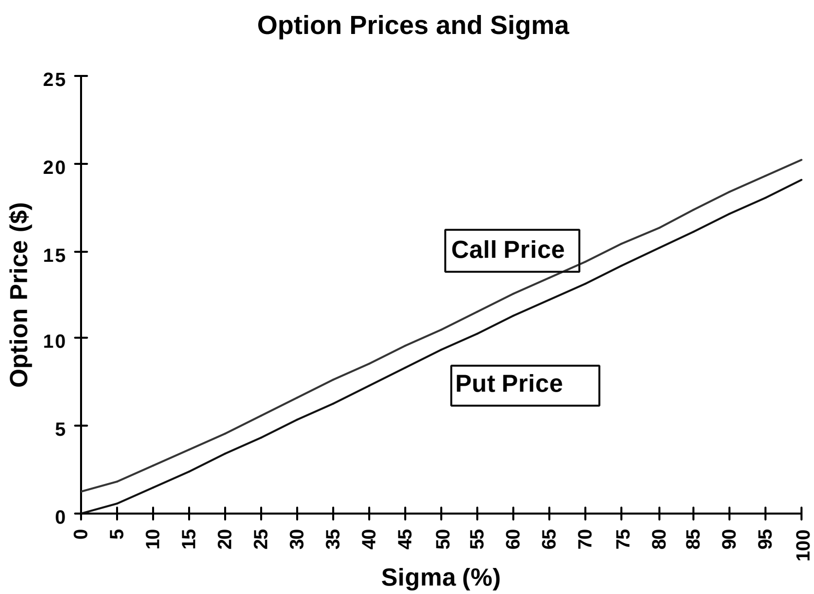
Factor Affecting Option Price	Greek	Call	Put
Stock Price (in dollars)	Delta	+	–
Strike Price (%)	Eta	>1	<1
Time until expiration	Theta		
Volatility (σ) (in percent)	Vega		
Risk-free rate (*r*)	Rho		
Div Yield (δ) (decrease stock price)			

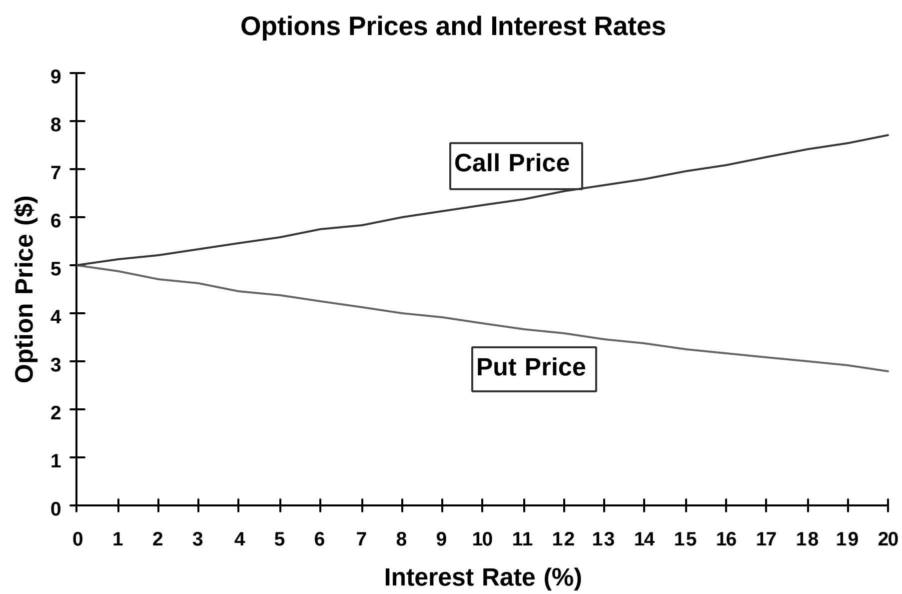
- Time to expiration – usually increases value of Euro options, but not always.
  -
- Increases in *r* –
  - decreasing value of put and call
  - increasing expected future stock price (reqd return) decreasing value of puts; increasing value of calls (dominates).
  - In practice, changes in *r* also affect stock prices, with ambiguous net effects.

Sensitivity of Option Prices (b)



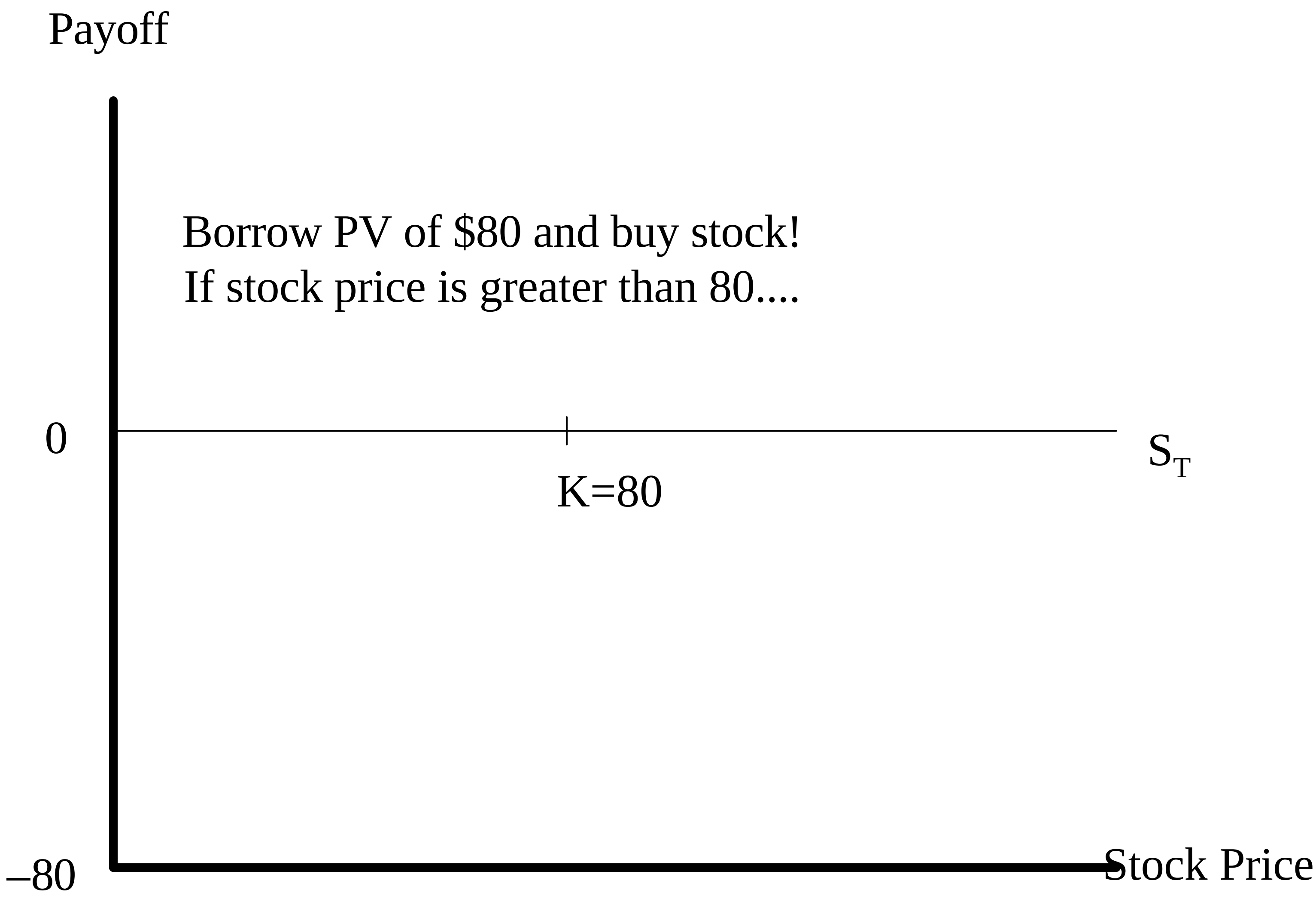






Put-Call Parity

Consider payoff from long call, short put, with K=80



Put-Call Parity

- What is value to investors of put and call options prior to expiration?

  Find two portfolios with same payoff.

  Then, by arbitrage, cost must be same.

Strategy 1: Consider payoff from long call and short put at same exercise price.

	$S_T \leq K$	$S_T > K$
Payoff from call owned		
Payoff from put written		
Total		

Strategy 2: Consider payoff from leveraged equity. Own stock and borrow Ke^{-rT}.

	$S_T \leq K$	$S_T > K$
Payoff from stock owned		
Payoff from loan		
Total		

Put-Call Parity

- Parity with Euro options – since payoffs equiv, cost of strategies must be equal!
  - Cost of [long call + short put] = cost of [leveraged equity]

    $c - p = S_0 - Ke^{-rT}$

    [call] = [protective put minus Ke^{-rT}]

- Parity with Euro option on div paying stock (cause lower *c* and greater *p*)

  $c - p = S_0 - \text{PV}(K) - \text{PV}(\text{Dividends})$

  $c - p = S_0 - Ke^{-rT} - D$

- Lower price bound on Euro option – setting *p*=0 or *c*=0 gives min value.

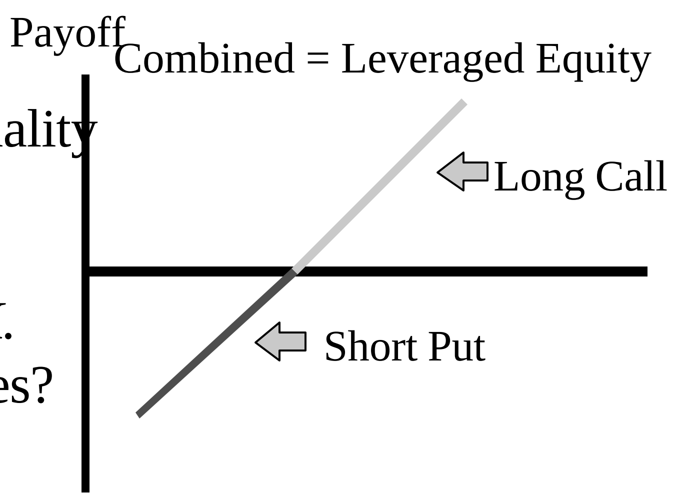
  $c >= S_0 - Ke^{-rT} - D$ (if *D*=0; call more valuable than levered equity!).

  $p >= D + Ke^{-rT} - S_0$

- American Options – we can only establish inequality

  $S_0 - K - D <= C - P <= S_0 - Ke^{-rT}$

- Look up near-dated options on *SPY* with same *K*.
  - What happens to *c*–*p* as *K* decreases/increases?
  - What happens to *c*–*p* as *T* increases?



Put-Call Parity: An Example

- Q: Are these fair prices for Euro calls and puts, given price of underlying stock?
  - S_0 = 110; D=0; r = 6%; c = 17; p = 5; T = 0.5 yr; K = 105
- A1:

.

- A2:

		CF(0)	CF(T)	Gross Return
Short (1)	Short call and long put			
Long (2)	Long asset; borrow PV(K)			

- If S_T > 105, deliver stock on call (+\$105), payoff loan (–\$105); put worthless.
- If S_T < 105, sell stock via put (+\$105), payoff loan (–\$105); call worthless.
  - In each case, earn \$3.90$e^{0.06*0.5}$.
- Verify put-call parity by looking up option prices on web.

Early Exercise

- Q: An American option is in-the-money. You have no inside info on future stock prices (efficient mkts). Should you exercise early?
  - Will payoff from exercise exceed the value of a European call?

- A1: Exercising an American call captures the intrinsic value = $S_0 - K$.
  By parity, value of Euro call is $c >= S_0 - Ke^{-rT} - D$.

  - If div are zero, intrinsic value less than value of Euro call!
    If D=0, American call will be not exercised early ($C = c$)**.**

  - If div large, intrinsic value may exceed value of Euro call, esp just prior to div
    If D>0, American call may (occasionally) be exercised early ($C \geq c$)**.**

- A2: American put can be exercised to capture intrinsic value = $K - S_0$.
  By parity, value of Euro put is $p >= D + Ke^{-rT} - S_0$.

  - Intrinsic value may exceed value of Euro put, esp if D is large, or if put is deep in-the-money (i.e., c -- from parity inequality -- is close to 0).
    American put may be exercised early ($P \geq p$).

Summary

- Factors affecting Option Prices
  - Strike price, time, volatility, risk-free rate, dividend yield

- Assumptions and Notation

- Upper and Lower Bounds for Options Prices

- Put-Call Parity
  - $c - p = S_0 - Ke^{-rT} - D$ or
  - $c - p = S_0 e^{-qt} - Ke^{-rT}$ for assets with continuous dividend yield

- Early Exercise
  - calls on div paying stock
  - puts on a non-div paying stock

- Effects of Dividends

- Empirical Research