Interest Rate Futures

- Introduction
  - Day count conventions

- T-bond Futures (long-term interest rate futures)
  - Mechanics
  - Interest rate risk and duration.
  - Using T-bond futures to hedge interest rate risk

- Eurodollar Futures (short-term interest rate futures)
  - Mechanics
  - Using ED futures to hedge interest rate risk.

Treasury Bond and Other Rate Quotes

- Treasury Bond quotes (interest usually paid semi-annually)
  - Cash price = Quoted spot price + Accrued Interest at Actual/Actual day count
  - 8% coupon bond quoted at 110, with coupons on Mar 1 and Sep 1.
  - Cash price on Jul 1 =

- Corporate Bond quotes (interest usually paid semi-annually)
  - Cash price = Quoted spot price + Accrued Interest at 30/360 day count.
  - 8% coupon bond quoted at 110, with coupons on Mar 1 and Sep 1.
  - Cash price on Jul 1 =

- T-bills – quoted on discount basis with actual/360 day count.
  - Quoted Interest Rate = $(100–Y)*360/n$ where Y is cash price.
  - 90-day T-bill trading at 97.50 has quoted rate =

- Eurodollar – quoted on add-on basis with actual/360 day count.
  - 3-mo ED deposit of $100,000 quoted at 10% earns $25,000 (2.5%) interest.
  - Eurodollar futures, however, are quoted on a discount basis!!

Treasury Bond Futures

- T-Bond futures (CBOT) – one of most actively traded futures contracts.
  - Deliver T-bonds with \$100,000 par, 15+ yrs maturity, not callable for 15 yrs.
  - Also several T-note futures contracts (~ 2-10 yrs maturity).
  - Expires in Mar, Jun, Sep, and Dec, extending about 2 yrs.
  - Last trading day -

- Delivery – actually delivery is rare, but range of bonds (30+) may be eligible.
  - Short position chooses which bond ("cheapest"), and payoff is adjusted.
  - Cash Flow (to short):

- Conversion factor – based on PV \$1 of principal on 1st day of delivery month.
  - Calculation currently assumes flat yield curve at 6% s.a.
  - Maturity and coupons rounded down to nearest 3-mo, with lag for 1st pymt.
  - If ylds > 6%, cheapest bonds are low-coupon, long maturity (high duration).

- Cash Flows satisfy: $F_0 = (S_0 – I)e^{rT}$ where I is PV of coupons during contract.
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Treasury Bond Futures Example 1

- Consider following T-bonds futures contract quoted at 93-08.
  - Quote of 93-08 means short receives $93,250 for delivery of $100,000 par.

- Q: Suppose you are short this futures contract and are considering delivering a 10% coupon bond maturing in 20 yrs and 60 days, with last coupon paid 123 days ago and next coupon in 60 days. The conversion factor is 1.4623.
  What price will you receive for delivering this bond?

- A2: Cash Flow (to short): Quoted futures price × Conv factor + Accrued Interest
  =

- Note that short position will deliver this bond if it has the best payoff.
  - I.e., Short position delivers bond that max [cash rec'd – cost on spot mkt]
  - [Quoted futures price×Conv + AI] – [Spot price + AI]

Treasury Bond Futures Example 2

- Q1: Suppose that the cheapest to deliver bond on a T-bond futures contract with delivery in 270 days is 12% coupon bond, currently quoted at $120. Last coupon was 60 days previous; next coupons are in 122 and 305 days. The conversion factor is 1.40. Term structure is flat at 10% c.c. over contract.
  - (1) Find cash price of bond: Cash price = Quoted spot price + Accrued Int.
  - (2) Find futures cash flow consistent with no-arbitrage: $F_0 = (S_0 – I)e^{rT}$
  - (3) Find quoted futures price: CF(to short) = QFP×Conv factor + Acc Int.

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CF:      6    (182 days)   6     (183 days)        6
Time:(–60)----(0)-------(122)--148days--(T=270)---(305)
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- A1:

- A2:

- A3: Given no-arbitrage cash flow, find quoted futures price:
  CF(to short) = Quoted Futures Price × Conv factor + Accrued Int (at T=270).

6 T-Bonds and Interest Rate Risk

- T-bonds futures contract – are often used to hedge "interest rate risk", or the response of bond prices to changing yields.
  - Interest rate risk often is measured by Duration (D*). (Excel: mduration)
  - Duration of a zero coupon bond is equal to its maturity.
  - Duration increases with maturity and decreases with coupon.
  - Effect on bond prices of change in yield is $\Delta B/B = -D^{*} \times \Delta y$ (c.c.)

- For each bond, find duration and change in bond price if yields increase 0.5%.
- (1) 2-yr zero coupon bond;
- A1:

- (2) 2-yr 8% coupon selling at par (YTM=8% s.a.);
- A2: D =

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-y t_i}}{B} \right]$$

- (3) 2 yr floating rate bond, with next coupon in 1 month.
- A3: D =

Yr (t)	CF	PV	PV(t)/B	t x PV(t)/B
0.5	40	38.46	0.0385	0.02
1	40	36.98	0.0370	0.04
1.5	40	35.56	0.0356	0.05
2	1040	889.00	0.8890	1.78
		1,000.00		**1.89**

7 Hedging Interest Rate Risk 1

- Many financial instituting are exposed to interest rate risk.
  - Bank assets (loans) have longer duration than bank liabilities (deposits). What happens when interest rates rise? What if durations were matched?
  - Fannie/Freddie assets (mortgages) have long duration.
    Plus, duration of assets (mortgages) varies with interest rates.
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- Duration matching - hedging of interest rate risk by matching durations of assets and liabilities. Aka bond portfolio immunization; "buy or sell duration".
  - Protects against small parallel shifts in the zero curve.
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- Hedging by duration – number of futures contracts required to hedge
  - $N^* = PD_P / F_c D_F$ where durations measured at maturity of hedge.
  - P is forward value of portfolio being hedged, with duration D_P
  - F_C is contract price for the interest rate futures contract, with duration D_F
  - .

Hedging Interest Rate Risk 2

- Q: Bond portfolio immunization. Suppose today is Aug 2. Fund mgr has \$10M in T-bonds and wishes to hedge value over the next three months. The portfolio's duration in 3-months will be 6.8 yrs.

  The fund mgr decides to hedge with Dec T-bond futures, with current futures price of 93-02 (93.0625 or \$93,062.50). The cheapest-to-deliver bond is expected to be a 20-yr, 12%/annum coupon bond, with a current yield of 8.8%/annum and duration of 9.2 yrs at maturity of the contract.

  How should this position be hedged?

- A1:

- A2: Number of contracts shorted is $N^* = PD_P / F_c D_F$

- Futures position will approx hedge portfolio against parallel shifts in rates! Falling prices will increase value of short hedge. Note: many financial institutions do simulations in addition to duration (plus convexity) calculations.

Eurodollars: Introduction

- Eurodollars - Deposits of U.S. dollars at foreign banks abroad or at foreign branch offices of U.S. banks or international banking facilities.
  - Typically short-term time deposits: Overnight to 1-yr; some up to 10 yrs.
  - Started in 1950's; gained popularity during 1980's interest rate caps.
  - Ex: Ford's UK subsidiary may keep accounts denom in $US, GBP, Yen.

- Eurodollar supply – generated when US entities pay abroad in $US, such as
  - purchases of goods and services
  - petroleum (OPEC accepts US$) and other raw materials.
  -

- Eurocurrency market **–** An int'l money market where bank deposits denominated in convertible currencies are traded. London is major hub.
  - ED are continually loaned, e.g., to US banks to satisfy reserve req.
  - Lent to non-financial firms to finance import-export, working capital, taxes.
  - LIBOR - Rate large int'l financial inst charge each other on ED deposits.
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Eurodollar Futures

- Eurodollar futures (3-mo) – futures contract on 3-mo ED time deposit.
  - Widely used by dealers in swaps, FRA and interest rate options to hedge.
  - Settled in cash, expires 3^{rd} Wed of delivery (Mar, Jun, Sep, Dec).
  - Maturities up to 10 yrs.
  -

- Eurodollar futures quotes – Mechanics are awkward, but cash settlement gives payoff similar to short position delivering 90-day T-bills, purchased at market price, and receiving futures price.
  - Value is 10,000[100–0.25(100–Q)], where Q is quoted price of contract.
  - At exp Q set to [100–R] where R = 3-mo ED rate q.c. annualized actual/360.
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- Suppose 3-mo ED rate is 10% at expiration (2.5% annualized by 360/90).
  - then quoted futures price is Q=

- Suppose the Dec 2005 3-mo ED contract is quoted at 90 (Q=90).
  - This means base (3-mo) price is 97.5, then scale to fraction of $1M.
  - Contract value is 10,000[100–0.25(100–90)] = .

Hedging Floating Rate Assets & Liabilities

- On Apr 29, a company borrows $15M for 3 months at floating rate of 1% over 1-mo LIBOR. The current 1-mo LIBOR rate is 8%. The Jun ED futures contract is quoted at 91.88 ($979,700) and Sep ED contract at 91.44 ($978,600).
  - How the company hedge against rising rates?

- Hedging requires going short some futures contracts for each payment, and closing the hedge when the corresponding loan payment is made. The interest rate on loan is reset every month.
  - So duration to hedge is 0.083 for each payment.
  - Each futures contract has duration D_F=0.25.
  - Recall $N^* = PD_P/F_cD_F$

- First month: rate is 8%+1%=9% or 0.75%/mo;
  - Payment is .

- Second month: $N^* = PD_P/F_cD_F =$

- Third month: $N^* = PD_P/F_cD_F =$

Interest Rate Futures Summary

- Treasury bond futures - Long-term interest rate contracts
  - Can be used to hedge value of bond portfolios with high duration.
  - Many eligible bonds to deliver. Short position calculates best option.

- Eurodollar future - Short-term interest rate futures contracts -
  - can be used to hedge value of bond portfolios with low durations

- Interest rate risk is often measured by duration.

- Duration – measure of interest rate risk that combines the effects of maturity, coupon rate and YTM. The duration of a zero bond is equal to its maturity.
  - Duration increases with maturity and decreases with coupon.