

1 Interest Rate Mkts and Forward Rates

- Interest Rates and Compounding
 - We use *continuous* compounding, for simplicity.
- Overview of Interest Rate Derivatives
 - Forward Rate Agreements, Swaps, Bond Futures, etc...
- Spot Rates – the YTM on zero coupon bonds.
 - Determining spot rates
- Forward Rates – the future zero (spot) rate implied by today's term structure.
 - Current 1-yr bonds and 2-yr bonds are linked by some implied YTM between years 1 and 2!
 - Example: suppose 1-yr spot rate is 10% and 2-yr spot rate is 10.5%. Then forward rate from year 1 to year 2 is about 11%!
 - Determining forward rates
- Forward Rate Agreements - an agreement that certain rate (R_K) will apply to a certain principal (L) during a certain future time period.
 - Valuing Forward Rates Agreements

2 Interest Rates and Compounding

- | APY Compounded | Periodic | Annual Rate (EAR) | FV of \$1 |
|----------------------|----------|-------------------------------------|-----------|
| • 12% Annual | | 12% | |
| • 12% Monthly | .12/12 | $(1+0.12/12)^{12} - 1 = 12.683\%$ | 112.683 |
| • 12% Daily | .12/365 | $(1+0.12/365)^{365} - 1 = 12.748\%$ | 112.748 |
| • R_m m | R_m/m | $(1+R_m/m)^m - 1$ | |
| • 12% Continuously | | $e^{0.12} - 1 = 12.750\%$ | 112.750 |
| • R_c Continuously | | $I*(e^{R_c/I} - 1)$ | |
- To calculate FV and PV, use e^{rT} instead of $(1+r)^T$ and e^{-rT} instead of $1/(1+r)^T$
 - To convert continuously compd rates (R_c) and rates compded m per yr (R_m).
 $R_c = m \ln(1 + R_m/m)$ and $R_m = m*(e^{R_c/m} - 1)$
 - Q: Consider 8%/yr c.c. Find equiv rates compd annually and quarterly.
 A1: $R_4 =$
 - Q: Consider 10%/yr with semi-annual compd. Find equiv rate with c.c.
 A1: $R_c =$

3 Overview of Interest Rate Derivatives

- Interest rate derivatives
 - Forward rate agreements
 - Swaps
 - T-bond futures (later)
 - ED futures and T-bill futures (later)

- Who hedges with interest rate derivatives?
 - manufacturers
 - depository institutions
 - commercial banks
 - thrifts/savings and loans (savings banks, credit unions).
 - other financial institutions
 - insurance companies, pension funds, mutual funds
 - investment banks issuing securities on firm commitment
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- Who speculates with interest rate derivatives? Usual suspects.
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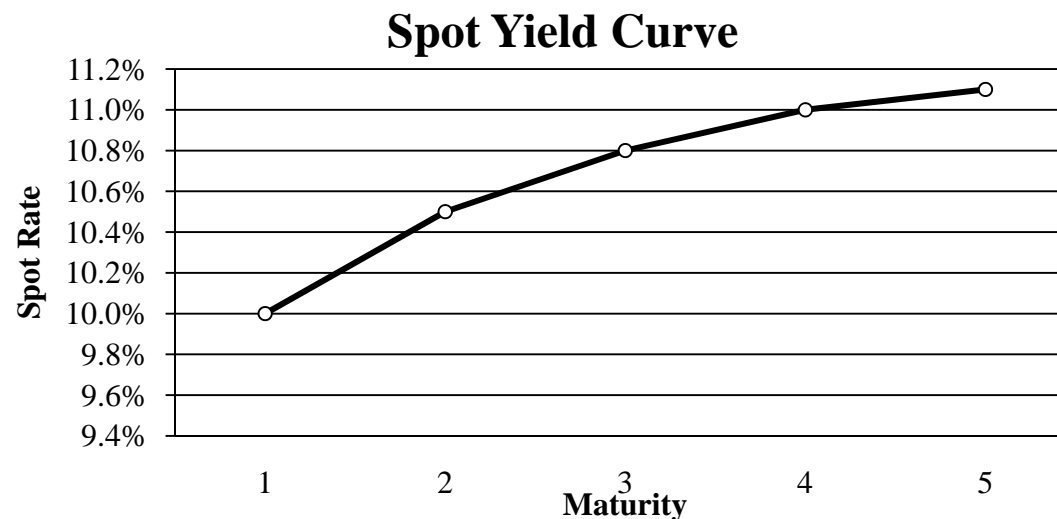
4 Treasury Bonds

- Treasury Bonds - Coupon bearing instruments, usually paid semi-annually.
 - Assumed to have no default risk. Maturities 2-10 yrs (notes) and 10-30 yrs.
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- T-bond Price quotes – price is quoted as % of par without accrued interest.
 - Cash price includes accrued interest.
 - Cash price = Quoted price (% par) + Accrued Interest (Actual/Actual)
 - 8% coupon bond quoted at 110, with coupons on Mar 1 and Sep 1.
 - Cash price on Jul 3 =
- Treasury bond pricing and YTM
 - price - obtained by discounting each cash flow at appropriate rate.
 - YTM - constant discount rate equating PV of cash flow with market price.
 - T-bonds with same maturity but different coupons have different YTM's!
 - E.g., 2-yr T-note with 6% coupon may have YTM of 10.48%.
 - But 2-yr T-note with 0% coupon may have YTM of 10.5%.

5 FRA: Bonds with Zero Coupons

- Spot rate – the YTM on a zero coupon bond. A very useful benchmark.
 - YTM on 2-yr coupon bond is weighted avg of 1-yr spot and 2-yr spot!
 - So, spot rate serves as “benchmark” interest rate at each maturity.
 - Spots rates can be used to discount any CF at that time with same risk.
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- Forward rate agreement (FRA) - an agreement that certain rate (R_K) will apply to a certain principal (L) during a certain future time period.
 - FRA’s can be used to lock-in future interest rate or hedge a future liability.
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6 FRA: Bond Pricing using Spot Rates

- Pricing coupon bonds with spot rates
 - We can discount each cash flow at the appropriate spot rate.
 - Q: What should be the price of a 2-yr bond with a 6% annual coupon?
 - A:
- Q: What is the YTM (c.c.) on this bond?
- A: Recall that YTM is *constant* discount rate equating price with PV of all CF.

$r =$

- Forward rate - future zero (spot) rate implied by today's term structure.
 - Ex: 10.5% (spot) over 2 yrs is same as 10% in first yr and f % in second.

Q1: Find the forward rate between years 1 and 2.

A1: $e^{0.105*2} =$

Q2: Find the forward rate between years 2 and 3.

A2: $e^{0.108*3} =$

T	Spot rate for n^{th} -yr	Forward rate for n^{th} yr
1	10%	
2	10.5%	11.0%
3	10.8%	11.4%
4	11%	11.6%
5	11.1%	11.5%

7 FRA: Locking in a Forward Rate

- Locking in forward rates – assume borrowing and lending rates are equal.
 - Forward rates can be locking in by buying and selling zero coupon bonds.

- Example Suppose you want to earn $f_{1,2} = 11\%$ beginning one year from now.
 - Borrow \$100 for 1 yr at 10% (or short a 1-yr zero bond).
 - Then invest for 2 yrs at 10.5% (or buy a 2-yr zero bond).
 - CF at yr 1: $-100 * e^{-0.10} =$
 - CF at yr 2: $+100 * e^{-0.105 * 2} =$
 - CF at yr 2 locks in forward rate of 11%, since $110.52 * e^{0.11} = 123.37$

	0	1	2
CF on Loan	+100		---
CF on 2-yr Zero Bond	-100	--	
Total CF	0		

- Moral – You can trade bonds to costlessly lock-in forward rates.
 - Hence, an forward rate agreement to pay current forward rate has no value.
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8 FRA: Forward Rate Agreements

- Forward rate agreement (FRA) - an agreement that certain rate (R_K) will apply to a certain principal (L) during a certain future time period.
 - In practice, FRA usually settled by exchanging PV of CF at T_1 .
 - Equivalent to exchanging interest at R_K and pay observed forward rate (R_F).
 - Let R = actual LIBOR rate and R_F = forward LIBOR rate between T_1 and T_2 .
- Valuing FRA – Value of FRA₂ with $R_K \neq R_F$ is PV of excess cash flows.

• Cash Flow on FRA (R_x has T_2-T_1 compounding)	T_1	T_2
FRA ₁	-L	+L(1+ $R_F(T_2-T_1)$).
Cash Flows on FRA ₂ earning $R_K > R_F$ is:	-L	+L(1+ $R_K(T_2-T_1)$).
Excess Cash Flow (FRA ₂ -FRA ₁)	0	+L($R_K - R_F$)(T_2-T_1)

- PV of Excess Cash Flow is $+L(R_K-R_F)(T_2-T_1)e^{-R_2T_2}$ where R_2 is cc spot rate at T_2
- Consider a FRA to receive 12% c.a. on \$1M beginning in one year and for one year. The current forward rate is 11.6278% c.a. and the 2-yr spot rate is 10.5%.
 - Value of FRA =

9 Interest Rate Market Summary

- Spot Rates – the YTM on zero coupon bonds.
- Forward Rates – the future zero (spot) rate implied by today’s term structure.
 - Current 1-yr bonds and 2-yr bonds are linked by some implied YTM between years 1 and 2! That implied YTM is the forward rate.
- Forward rate agreement (FRA) - an agreement that certain rate (R_K) will apply to a certain principal (L) during a certain future time period.
 - Usually settled by exchanging PV of CF at T_1 .
 - The value of a FRA is just the PV of the future cash flows.
 - Can be used to “lock-in” a particular future interest rate.