Hedging Strategies Using Futures (Part II)

- Introduction and Exmples
  - What is hedging? Why hedge? Examples. Should firms hedge?

- Hedging and Basis Risk
  - mismatch between contract termination and hedge horizon.
    Mismatch between contract specification and commodity being hedged.

- Hedging with stock index futures
  - Example with mismatch in hedge horizon.
  - Hedging "beta risk" (end of part I)
  - Cross hedge – mismatch with contract specification.
    - Minimum Variance (optimal) Hedge Ratio
    - Example with Stock Index Futures

- Other issues
  - Creating synthetic positions.
  - Rolling the hedge forward

- See also http://www.hedgestreet.com
  - –Allows trading of variety of hedgelets with payoff $10

Cross Hedges – Matching Futures Contract

- **Recall:** Q: Suppose the portfolio's objective is to track the SP500. Currently, the portfolio beta is 1.0, with $5M in assets and SP500 futures price is 1,010.
  - What position in futures contracts on SP500 hedges the portfolio?

- A1: Number of (short) contracts = $5M / (250*1010) ≈ $5M / $0.25M = 20.
  - Changes in hedge will offset changes in portfolio at maturity of futures.

- Q: Suppose now the portfolio beta is 1.5 with $5M in assets.
  - What position in futures contracts on S&P 500 hedges the portfolio?
  - Probably want more than 20 contracts. But how many?
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- Hedge ratio –
  - Cross hedge – asset price being hedged differs from asset underlying futures.
  - If futures contract matches commodity and terminates with hedge, ratio is 1.
  - Otherwise, optimal hedge ratio may not be 1.
  - Ex: hedge price of jet fuel with futures on crude (but spread varies).
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Cross Hedges – Optimal Hedge Ratio

- Optimal hedge ratio (h) - Proportion of exposure that minimizes variance of change in value of hedger's position (ΔV). Note $|\Delta V| = |\Delta S - h\Delta F|$.
  - Let σ_S is std dev of ΔS (change in spot price). *Excel: =stdev(..)*
  - Let σ_F is std dev of ΔF (change in futures price).
  - Let ρ be correlation between ΔS and ΔF. *Excel: =Correl(..)*
  - Min(h) var(ΔV) = $\sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$

- Optimal number of contracts (N^*).
  - Q_A size of position hedged (units); Q_F size of futures contract (units).

$$h^* = \rho \frac{\sigma_S}{\sigma_F} = \frac{\text{cov}(\Delta S, \Delta F)}{\text{var}(\Delta F)} \qquad N^* = h^* \frac{Q_A}{Q_F} \qquad N^* = \beta \frac{\$P}{\$F}$$

- Equity portfolios -
  - For equity portfolios, ΔS and ΔF are usually defined as <u>percent</u> changes.
  - Optimal hedge ratio is given by β of equity portfolio ($h^* = \beta$).
  - For other commodities, hedge ratio can be thought of as "beta" vs contract.
  - Tailing hedge –

- Hedge Effectiveness – R^2 from regression of ΔS on ΔF, which is ρ^2.
- Time interval for ΔS – .

Hedging wwith Equity Index Futures 1

- Q: Suppose portfolio beta is 1.5 with \$5M in assets and SP500 futures are 1,010.
  - What position in SP500 futures contracts hedges the portfolio?
- A: N^*=

- Q: Suppose risk-free rate is 4%; div yld is 1%; SP500 is 1,000.
  - Futures price for delivery in 4-mo is ≈ 1010.
  - After 3-mo, suppose SP500 is 900, and new futures price is 902.
  - Find value of hedged portfolio after 3-months.

- A1: Gain on short futures:

- A2: Use CAPM: $E(R_i)–R_f = \beta\ [E(R_m)–R_f]$ to estimate value of portfolio
  Total return on mkt is (incl 3-mo div) =
  Total return on equity port: $E(R_i)$ =
  Value of equity portfolio is (incl div) =

- A3: Value of hedged portfolio =

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Hedging with Equity Index Futures 2

		Annual c.c.	Period c.c.	Period a.c.			
	R_f	4.00%	1.00%	1.01%	T_2-T_1	0.25	
	Div Yld	1.00%	0.25%	0.25%	Maturity F_0	0.33	
	S_1	1,000			$F_1=S_1e^{(r-q)T}$	1,010.05	
	Portfolio	$5,000,000			Multiplier	250	
	β_p	1.5			Contracts	30	
S_2	F_2	Cash Flow on Short Hedge $N*Q(F_1$-$F_2)$	Periodic Return on Mkt Incl Div R_m	Periodic Return on Portfolio (CAPM) Incl Div E(R_p)	Cash Flow on Equity Portfolio	Cash Flow on Hedged Portfolio	Return on Hedged Portfolio
800	802.00	1,560,357	-19.75%	-30.13%	-1,506,352	54,006	1.08%
900	902.25	808,480	-9.75%	-15.13%	-756,352	52,128	1.04%
1000	1,002.50	56,603	0.25%	-0.13%	-6,352	50,251	1.01%
1100	1,102.75	-695,275	10.25%	14.87%	743,648	48,373	0.97%
1200	1,203.00	-1,447,152	20.25%	29.87%	1,493,648	46,496	0.93%

- Will hedge actually this work this well? Will cash flows be as predicted?
  - Recall: beta may not perfectly capture return on portfolio (unsystematic risk).
  - Variation in risk-free (and div yld) may cause variation in F_2.

6 Hedging with Equity Index Futures 3

- Q: Suppose you work for MSFT and have restricted stock. You are concerned about adverse reactions to antitrust issues. Can you use SP500 futures to hedge?
  OR
  Q: Suppose you worked for PCLN in 1999 and had tons of restricted stock. You were concerned about "dot.com" bubble bursting. How might this be hedged?

- A:

- Changing the equity portfolio's beta: $N^* = |\beta - \beta^*| \times P/F$
  - Previous example created hedged portfolio with $\beta^*=0$.
  - To decrease beta, short futures. To increase beta, go long futures.
  - Note: Target N^* solves: $\beta^* = \beta P/P + \beta_f N^* F/P$ assuming $\beta_f = 1$.
    where mkt value of stock + futures is P, since marked futures value is zero

- Q: What position is necessary to increase portfolio beta to 2.0?
- A:

- Q: What position is necessary to reduce the portfolio beta to 0.75?
- A:

7 Synthetic Equity Portfolios

- Q: Suppose you manage a mutual fund and have a large inflow of cash ($10M) that you want to invest quickly and cheaply before your investment committee next meets to hammer out individual investments. You would be satisfied if, in the interim, you could track SP500 (futures price is 900). What might you do?

- A: Invest in T-bills and futures, using: $N^* = |\beta - \beta^*| \times P/F$
  - SP500 has $\beta^*=1$; T-bills have $\beta=0$;
  - $N^*=$
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- Hedging Note:
  - Expiration should be soon *after* end of hedge, with no risk of delivery.
  - Futures price should be highly correlated with hedge.

- Hedging Note:
  - Strip hedge – long constant, matched futures each month. (Liquidity risk.)
  - Stack hedge – choose contract with single maturity to hedge PV of liability.
  - Stack and roll hedge – roll over liquid short-term contracts. (Basis risk.)

Hedging Notes I

- MGRM – large industrial conglomerate in metals, mining and engineering.
  - 1993 $16B in sales; $10B in assets. 15 major subsidiaries.
  - MGRM sold 154M bbl oil to retailers at fixed prices for up to10-yrs.
  - Planned to buy oil on spot market. Exposed to risk of higher future spot price.
- Hedging options.
  - Opt 1 – Buy strip of forwards, matching price and quantity. But illiquid.
  - Opt 2 – Roll over short-term futures. MGRM long contracts for 154M bbl.
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- Problems
  - Futures prices were above spot contributing to rollover losses.
  - Oil plummets 30%. Futures down; Margin calls; rollover losses $50M/mo
  - MGRM closes futures with losses of $1.3B. Mgt fired.
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- China Aviation Oil (Singapore) bets oil prices will fall in 2004. Loses $550M
  - Prices rose $30/bbl, $40, $50; Draw credit; Violates margin.…
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Hedging Summary

- Futures can be used to hedge risks
  - Long futures if hedger needs to purchase commodity.
  - Short futures if hedger needs to sell commodity.

- Basis Risk – Many hedges are not "perfect".
  - Hedging horizon does not match contract termination (basis risk).
  - Asset being hedged may not match contract specs (basis risk/cross hedge).

- Choosing best futures contract for a hedge
  - Futures price should be highly correlated with hedge.
  - Choose expiration soonest *after* end of hedge; Or roll over short term contracts.
  - Rolling over short term contracts may be more liquid.
  - Data: SP500 Div yld (SP); SP500 futures (WSJ); LIBOR (WSJ)

- Optimal hedge ratio - Proportion of exposure that should optimally be hedged.
  - Optimal hedge ratio on equity portfolio is beta (if futures contract is index).

$$N^* = h^* \frac{Q_A}{Q_F} \qquad N^* = \left|\beta - \beta^*\right| \frac{\$P}{\$F} \qquad h^* = \rho \frac{\sigma_S}{\sigma_F} = \frac{\operatorname{cov}(\Delta S, \Delta F)}{\operatorname{var}(\Delta F)}$$