Finance with Dr. John Elder / Formula Sheet for Derivative Securities

Discounting Cash Flows:

Lump-Sum Payments with constant "r" (a common simplification of the basic concept):

FV of a payment "C" in "T" years = $C(1+r)^T$ or Ce^{rT} with continuous compounding

PV of a payment "C" to be received in "T" years = $C/(1+r)^T$ or Ce^{-rT} with continuous compounding

Annuities with constant "r" (an annuity is just a sequence of lump-sum payments):

PV of an annuity payment "C" for "T" years, growing at rate "g", and beginning in 1 year =

$$PV = C\left[\frac{1}{(1+r)} + \frac{(1+g)}{(1+r)^2} + \cdots + \frac{(1+g)^{T-1}}{(1+r)^T}\right] = C\,\frac{1-\left[\frac{1+g}{1+r}\right]^T}{r-g} \text{ or } C\,\frac{1-\left[\frac{e^g}{e^r}\right]^T}{e^{(r-g)}-1} \text{ c.c.}$$

Perpetuities with constant "r" (a perpetuity is an annuity that goes on forever):

PV of a perpetuity "C" forever, beginning in 1 year, and with grows at rate "g" = $C/(r-g)$ or $C/(e^{(r-g)}-1)$ c.c.

Interest Rates: R_m (APR) is annual rate compounded over m periods; R_c is continuous compounded rate per year.

EAR = $(1+R_m/m)^m - 1$ $\qquad R_c = m*\ln(1+R_m/m)$ $\qquad R_m = m*(e^{Rc/m}-1)$

Futures:

Notation: S_0: Spot price today; F_0: Futures/forward price today; T: Time until delivery; r: risk-free rate at maturity T

Futures pricing: $F_0 = (S_0 - I)\,e^{(c-y)T}$ where and c is cost of carry; y is convenience yield; I is PV of income.

Investment asset: convenience yield is zero (y=0).

Asset with known yield: $c = r - q$ where q is avg unit yield. FX: $c = r - r_{for}$ where r_{for} is foreign risk-free rate.

Commodities: $c = r + u$ where u is storage cost/unit time as % of asset value; or $I = -U$ where U is PV of storage cost.

Value of long forward: $f = (F_0 - K)e^{-rT}$ where K is delivery price; F_0 is forward price that would apply today.

Hedging:

N^* optimal number of contracts; N_A size of position being hedged (units); Q_F size of one futures contract (units).

P is forward value of interest rate portfolio, with duration D_P measured at maturity of hedge.

F_C is contract price for interest rate futures contract, with duration D_F.

Optimal hedge ratio $h^* = \rho\dfrac{\sigma_S}{\sigma_F} = \dfrac{\text{cov}(\Delta S, \Delta F)}{\text{var}(\Delta F)}$;

Commodities: $N^* = h^*(Q_A/Q_F)$ $\qquad$ Equities: $N^* = |\beta - \beta^*|(\$P/\$F)$ $\qquad$ Bonds: $N^* = PD_P / F_c D_F$

Interest Rate Markets and Futures:

Forward rate agreement to earn R_K and pay R_F on principal L: $= L(R_K - R_F)(T_2 - T_1)e^{-R2T2}$ where R_2 is spot rate at T_2.

Treasury bond Futures: Cash rec'd by short = Quoted futures price×Conv factor + Accrued interest

Conv factor is PV of $1 of principal assuming yield curve flat at 6% compounded semi-annually

Bond maturity and time until coupons rounded down to nearest 3 months.

Treasury bond quote: Cash price = Quoted price + Accrued Interest at Actual/Actual day count.

Corporate bond quote: Cash price = Quoted price + Accrued Interest at 30/360 day count.

T-bill quote: discount basis, with *rate* = $(100-Y)*360/n$ where Y is cash price, and actual/360 day count.

Eurodollar quote: add-on basis, with *rate* = $((100-Y)/Y)*360/n$, where Y is cash price and actual/360 day count.

Statistics:

$$Mean: E(R) = \sum_{i=1}^{N} P_i R_i \;\; where\; P_i = \frac{1}{N} \;\; for\;\; sample \qquad Variance: \sigma^2 = \sum_{i=1}^{N} P_i\big(R_i - E(R)\big)^2 \;\; where\; P_i = \frac{1}{N-1} \;\; for\; sample$$

$$Covariance: \sigma_{1,2} = \sum_{i=1}^{N} P_i\big(R_{i,1} - E(R_1)\big)\big(R_{i,2} - E(R_2)\big) \qquad Correlation: \rho_{1,2} = \frac{\sigma_{1,2}}{\sigma_1 \sigma_2}$$

$$Mean\ of\ portfolio: E(R_p) = aE(R_1) + (1-a)E(R_2) \qquad Var\ of\ portfolio: \text{var}(R_p) = a^2\sigma_1^2 + (1-a)^2\sigma_2^2 + 2a(1-a)\sigma_1\sigma_2\rho_{1,2}$$

Asset Pricing:

CAPM: $\text{E}(R_i) - R_f = \beta_i \times [\text{E}(R_M) - R_f]$ $\qquad$ where $\beta_i = \text{cov}(R_i, R_m)/\text{var}(R_m)$

Options:

Notation:

c and p European options; C and P American options; D: PV of dividends; K: exercise price

Put-Call Parity Relationships:

Put-Call Parity for Euro Options: $c - p = S_0 - Ke^{-rT} - D$; $\qquad$ American Options: $S_0 - K - D \le C - P \le S_0 - Ke^{-rT}$

Put-Call parity with constant div yield $c - p = S_0 e^{-qT} - Ke^{-rT}$

Binomial Model: $\Delta = (f_u - f_d)/(S_0 u - S_o d)$; $\qquad$ Risk-neutral: $f = (pf_u + (1-p) f_d)*e^{-r(dt)}$ where $p = (e^{r(dt)} - d)/(u-d)$.

Black-Scholes: $c = S_o e^{-qT} \text{N}(d_1) - \text{K}\, e^{-rT} \text{N}(d_2)$ where $d_1 = [\ln(S_0/K) + (r - \delta + \sigma^2/2)\text{T}] / (\sigma \text{T}^{1/2})$; $\quad d_2 = d_1 - \sigma \text{T}^{1/2}$

Delta-neutral portfolio: $N_T^* = -\Delta_p/\Delta_T$ $\qquad$ Profits on delta-neutral portfolio: $\partial\Pi \approx \Theta\partial t + ½\Gamma\partial S^2$.

Gamma-neutral portfolio: $N_T^* = -\Gamma_p/\Gamma_T$